Stat 513
Emerson, Winter 04
Homework #7
March 3, 2004

Supplemental written problems due Monday, March 8, 2004 at the beginning of class.

Let $X_i, i = 1, \ldots, n$ be i.i.d. random variables with mean μ and variance $\sigma^2 < \infty$, and let $Y_i, i = 1, \ldots, m$ be i.i.d. random variables with mean ν and variance $\tau^2 < \infty$. We shall assume that as $n \to \infty$, the ratio $n/(m+n) \to \lambda$.

1. Suppose the X_i's are normally distributed. Use Basu's theorem to prove that the sample mean $\overline{X}$ is independent of the sample variance s_X^2. (Hint: Consider first the case where σ^2 is known. Examine the complete sufficient statistic for μ and the distribution of s_X^2. Then argue that the independence of the sample mean and the sample variance will not be altered by the lack of knowledge of σ^2.)
2. Now suppose that the X_i's have the Bernoulli distribution. Show that the sample mean and sample variance are not independent in this problem.
3. Suppose the X_i's and Y_i's are normally distributed.
   a. Find the likelihood ratio test for $H_0 : \mu = \nu$ versus $H_1 : \mu \neq \nu$ when $\sigma^2 = \tau^2$.
   b. Show that the Wald and score tests are equivalent to the LR test in this problem.
   c. What is the small sample distribution for the test in part a? What is the critical value of a level α test?
4. Now consider the general nonparametric problem (i.e., we only know the means and variances, not the parametric distribution).
   a. What is the asymptotic distribution of the test statistic you derived in problem 3? Do not presume that the variances are equal for this problem.
   b. Show that the test you derived in problem 3 is not necessarily level α when testing $H_0 : \mu = \nu$ versus $H_1 : \mu \neq \nu$. Show that it is level α when testing $H_0^* : \mu = \nu$ AND $\sigma^2 = \tau^2$ versus $H_1^* : \mu \neq \nu$ OR $\sigma^2 \neq \tau^2$.
   c. Show that the test you derived in problem 3 is not consistent when testing $H_0^* : \mu = \nu$ AND $\sigma^2 = \tau^2$ versus $H_1^* : \mu \neq \nu$ OR $\sigma^2 \neq \tau^2$.?