Stat 513
Emerson, Winter 04
Homework #5
February 13, 2004

Supplemental written problems due Friday, February 20, 2004 at the beginning of class.

1. Let X_{ijt} be the measurement made at time $t = 0, 1$ on the jth subject ($j = 1, \ldots, n$) in treatment group $i = 0, 1$ in some randomized experiment. Individuals in the two treatment groups are independent, and measurements made on distinct individuals in each treatment group are also independent. Measurements made at different times on the same individual are jointly normal and correlated with correlation ρ. Due to randomization, we regard the measurements at time $t = 0$ to be identically distributed, with mean μ_0 and variance σ^2. Following treatment, we assume that measurements in group $i = 0$ have mean μ_1 and variance σ^2, and measurements in group $i = 1$ have mean $\mu_1 + \Delta$ and variance σ^2. (Δ thus measures the effect of treatment on the measurements.) Presuming that σ^2 and ρ are known, find the uniform minimum variance unbiased estimate (UMVUE) of Δ. What is its distribution? Does it meet the Cramér-Rao lower bound for estimates of Δ?

2. We consider a sequential experiment in which we have potential observations X_1 and X_2 which are independent and identically distributed $X_i \sim \mathcal{B}(3, p)$. Our sequential sampling plan is as follows: We observe X_1, and if $X_1 > 1$, we stop. Otherwise we continue sampling to observe X_2. At the end of our experiment, we have the bivariate sequential test statistic

$$(M, S) = \begin{cases} (1, X_1) & \text{if } X_1 \leq a = -1 \text{ or } X_1 \geq b = 2 \\ (2, X_1 + X_2) & \text{otherwise.} \end{cases}$$

   Note that $X_1/3$ is an unbiased estimate of p, but (as you might expect from Homework 3 from last quarter) $S/(3M)$ is biased. Use the minimal sufficient statistic for p and the Rao-Blackwell Improvement Theorem to find an unbiased estimator with smaller variance than X_1. Provide explicit values of this estimator for each possible value of (M, S). Is this estimator the UMVUE? Explain your answer.