Stat 513
Emerson, Winter 04
Homework #3
January 23, 2004

Supplemental written problems due Friday, January 30, 2004 at the beginning of class.

1. Let Y_i, $i = 1, \ldots, n$ be independent exponential random variables with $Y_i \sim \mathcal{E}(\log(2)/\theta)$ (so $F_Y(y) = 1 - \exp(-\log(2)y/\theta)$ for $0 < y < \infty$.
   a. Find the parametric MLE of the median of the distribution of Y_i. Derive its asymptotic distribution.
   b. Find the asymptotic distribution of the sample median.
   c. What is the asymptotic relative efficiency of the two estimators found in parts 1a and 1b?
   d. Now suppose that the true distribution of the independent Y_i's is as lognormal $Y_i \sim LN(\mu, \sigma^2)$, having density
   $$f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma y} \exp\left(-\frac{(\log(y) - \mu)^2}{2\sigma^2}\right) \mathbf{1}_{[y>0]}.$$
   Further suppose $\mu = \log(\theta)$. For what function of θ is the estimator you found in part a consistent? What is the asymptotic distribution of the estimator from part 1a under this new distribution for Y?
2. Let Y_i, $i = 1, \ldots, n$ be independent lognormal random variables with $Y_i \sim LN(\log(\theta), \sigma^2)$.
   a. Find the parametric MLE of the median of the distribution of Y_i. Derive its asymptotic distribution.
   b. Find the asymptotic distribution of the sample median.
   c. What is the asymptotic relative efficiency of the two estimators found in parts 2a and 2b?
   d. Now suppose that the true distribution of the independent Y_i's is as exponential $Y_i \sim \mathcal{E}(\log(2)/\theta)$ as in problem 1. For what function of θ is the estimator you found in part 2a consistent? What is the asymptotic distribution of the estimator from part 2a under this new distribution for Y?
3. Discuss the relative merits of using parametric versus nonparametric estimators relative to your results in problems 1 and 2.