Stat 512
Emerson, Fall 03
Midterm #1
October 29, 2003

Supplemental problems to be handed in at the beginning of class on Wednesday, November 5.

Problems 1 through 3 refer to the following probability model.

Let X_{ijt} be the measurement made at time $t = 0, 1$ on the jth subject ($j = 1, \ldots, n$) in treatment group $i = 0, 1$ in some randomized experiment. Individuals in the two treatment groups are independent, and measurements made on distinct individuals in each treatment group are also independent. Measurements made at different times on the same individual are correlated with correlation ρ. Due to randomization, we regard the measurements at time $t = 0$ to be identically distributed, with mean μ_0 and variance σ^2. Following treatment, we assume that measurements in group $i = 0$ have mean μ_1 and variance σ^2, and measurements in group $i = 1$ have mean $\mu_1 + \Delta$ and variance σ^2. (Δ thus measures the effect of treatment on the measurements.)

1. Let $\overline{X}_{i\cdot 1} = \sum_{j=1}^{n} X_{ij1}/n$, and let $\hat{\Delta} = \overline{X}_{1\cdot 1} - \overline{X}_{0\cdot 1}$. *(In this problem, we are ignoring the baseline value measured at time 0.)*
   a. Find $E[\hat{\Delta}]$.
   b. Find $Var(\hat{\Delta})$.
2. Let $D_{ij} = X_{ij1} - X_{ij0}$, $\overline{D}_{i\cdot} = \sum_{j=1}^{n} D_{ij}/n$, and let $\hat{\Delta} = \overline{D}_{1\cdot} - \overline{D}_{0\cdot}$. *(In this problem, we are analyzing the difference between the final measurement made at time 1 and the baseline value measured at time 0.)*
   a. Find $E[D_{ij}]$.
   b. Find $Var(D_{ij})$.
   c. Find $E[\hat{\Delta}]$.
   d. Find $Var(\hat{\Delta})$.
3. Let $W_{ij} = X_{ij1} - \rho X_{ij0}$, $\overline{W}_{i\cdot} = \sum_{j=1}^{n} W_{ij}/n$, and let $\hat{\Delta} = \overline{W}_{1\cdot} - \overline{W}_{0\cdot}$. *(In this problem, we are adjusting the the final measurement made at time 1 for a portion of the baseline value measured at time 0.)*
   a. Find $E[W_{ij}]$.
   b. Find $Var(W_{ij})$.
   c. Find $E[\hat{\Delta}]$.
   d. Find $Var(\hat{\Delta})$.
   e. Contrast the mean and variance of the estimator derived in this problem with

those derived in problems 1 and 2.