Stat 512
Emerson, Fall 03
Homework #3
October 17, 2003

Supplemental written problems due Wednesday, October 22, 2003 at the beginning of class.

1. We consider a sequential experiment in which we have potential observations X_1 and X_2 which are independent and identically distributed $X_i \sim \mathcal{N}(\mu, \sigma^2)$. Our sequential sampling plan is as follows: We observe X_1, and if, for some prespecified $a < b$, $X_1 \leq a$ or $X_1 \geq b$, we stop. Otherwise we continue sampling to observe X_2. At the end of our experiment, we have the bivariate sequential test statistic

$$(M, S) = \begin{cases} (1, X_1) & \text{if } X_1 \leq a \text{ or } X_1 \geq b \\ (2, X_1 + X_2) & \text{otherwise.} \end{cases}$$

In order to estimate the unknown mean μ, we use the observed sample mean $\hat{\mu} = S/M$.

a. Find the density for (M, S). (This cannot be solved in closed form, so it is sufficient to write down the integral you would use to find it.)

b. Suppose $\mu = 0$, $\sigma^2 = 1$, $a = 0$, $b = 2.7897$.

   i. Find $Pr[M = 1, S \leq a]$.

   ii. Find $Pr[M = 1, S \geq b]$.

   iii. Find $Pr[M = 2, S \leq 0]$.

   iv. Find the value of c such that $Pr[M = 1, S \geq b] + Pr[M = 2, S \geq c] = 0.025$

c. Derive a formula for the expected value for $\hat{\mu}$ in the general case. Under what conditions will $E[\hat{\mu}] = \mu$? If an estimator $\hat{\mu}$ satisfies $E[\hat{\mu}] = \mu$, $\forall \mu$, we call that estimator unbiased. Under what conditions on our sampling plan is $\hat{\mu}$ unbiased?