Biost/Stat 533
Emerson, Spr 09
Homework #6
June 1, 2009

Written problems to be handed in Friday, June 5 at 9 am.

All problems consider the general parametric regression model in which we observe pairs $(Y_i, \vec{X}_i)$ for $i = 1, \ldots, n$ in which

$$Y_i \mid \vec{X}_i \sim f_Y(y; \theta_i) \qquad \text{with} \qquad g(\theta_i) = \vec{X}_i \vec{\beta},$$

with the Y_i's mutually independent, $\vec{X}_i = (1, X_{i1}, X_{i2}, \ldots, X_{ip}$ known covariates, and $\vec{\beta}$ a $p+1$ vector to be estimated and/or tested.

1. Suppose the link function g is the identity function $g(x) = x$. Find the score equations for the following choices of f_Y and θ_i.
   a. Bernoulli: $Y_i \sim \mathcal{B}(1, p_i)$ and $\theta_i = p_i$.
   b. Poisson: $Y_i \sim \mathcal{P}(\lambda_i)$ and $\theta_i = \lambda_i$.
   c. Exponential: $Y_i \sim \mathcal{E}(\lambda_i)$ and $\theta_i = \lambda_i$, where $E(Y_i) = \lambda_i$.
   d. Exponential: $Y_i \sim \mathcal{E}(\lambda_i)$ and $\theta_i = \lambda_i$, where $E(Y_i) = 1/\lambda_i$.
2. Repeat problem 1 with the specified link functions.
   a. Bernoulli: $Y_i \sim \mathcal{B}(1, p_i)$ and $\theta_i = p_i$. Use link $g(x) = logit(x)$.
   b. Poisson: $Y_i \sim \mathcal{P}(\lambda_i)$and $\theta_i = \lambda_i$. Use link $g(x) = \log(x)$.
   c. Exponential: $Y_i \sim \mathcal{E}(\lambda_i)$ and $\theta_i = \lambda_i$, where $E(Y_i) = \lambda_i$. Use link $g(x) = \log(x)$.
   d. Exponential: $Y_i \sim \mathcal{E}(\lambda_i)$ and $\theta_i = \lambda_i$, where $E(Y_i) = 1/\lambda_i$. Use link $g(x) = \log(x)$.
   e. Exponential: $Y_i \sim \mathcal{E}(\lambda_i)$ and $\theta_i = \lambda_i$, where $E(Y_i) = \lambda_i$. Use link $g(x) = 1/x$.
   f. Exponential: $Y_i \sim \mathcal{E}(\lambda_i)$ and $\theta_i = \lambda_i$, where $E(Y_i) = 1/\lambda_i$. Use link $g(x) = 1/x$.
3. Comment on the similarity of the forms of these score equations for exponential family models.