**(1)**

**Methods:** In the analysis below, we use the fact that the sample means are asymptotically normally distributed to justify our use of approximate Z statistics based on the values reported in the table. We form our 95% CIs using a critical value of 1.96 based on the Z-distribution, which has 2.5% of its density to the right of 1.96 and 2.5% of its density to the left of -1.96. We use pooled standard error estimates when estimating the difference in mean cholesterol between males and females in our data. We define our efficiency weights as being inversely proportional to the squared standard error estimate in each stratum. Our importance weights will be equal to the sample frequency of each stratum in the data.

**a.** We find that the mean cholesterol for Caucasian males in our data is 25.3 mg/dL lower than the mean cholesterol level of the Caucasian females in our data. The associated standard error for this estimate is 1.55 mg/dL, yielding a 95% CI of [-28.3 mg/dL, -22.3 mg/dL]. That is, our observed data are consistent with mean cholesterol truly being between 28.3 mg/dL lower and 22.3 mg/dL lower for elderly Caucasian males than for elderly Caucasian females. The accompanying two-sided p-value for a test of the true difference in means between these two groups being 0 is less than$10^{-3}$, so we have evidence of an association between cholesterol and sex among elderly Caucasians.

**b.** We find that the mean cholesterol for the Non-Caucasian males in our data is 15.7 mg/dL lower than the mean cholesterol level of the Non-Caucasian females in our data. The associated standard error for this estimate is 3.45 mg/dL, yielding a 95% CI of [-22.50 mg/dL, -8.94 mg/dL]. That is, our observed data are consistent with mean cholesterol truly being between 8.94 mg/dL lower and 22.5 mg/dL lower for elderly Non-Caucasian males than for elderly non-Caucasian females. The accompanying two-sided p-value for a test of the true difference in means between these two groups being 0 is less than$10^{-3}$, so we have evidence of an association between cholesterol and sex among elderly Non-Caucasians.

**c.**

* *Importance weights:* Using the relative frequencies in our sample, we weight the Caucasian stratum by 0.8405 and the Non-Caucasian stratum by 0.1595. After adjustment for race, we estimate that mean cholesterol for males is 23.77 mg/dL lower than mean cholesterol for females. The associated standard error for this estimate is 1.41 mg/dL, yielding a 95% CI of [-26.5 mg/dL, -21.0 mg/dL]. That is, our observed data are consistent with mean cholesterol truly being between 26.5 mg/dL lower and 21 mg/dL lower in men relative to women, after accounting for race. The accompanying two-sided p-value testing whether the difference in mean cholesterol between males and females (after accounting for race) is less than $10^{-3}$, so we have evidence of an association between cholesterol and sex among elderly individuals, after controlling for race.
* *Efficiency weights:* Here, we attach a weight of 0.832 to the Caucasians stratum and a weight of 0.168 to the Non-Caucasian stratum. After adjustment for race, we estimate that mean cholesterol for males is 23.7 mg/dL lower than mean cholesterol for females. The associated standard error for this estimate is 1.41 mg/dL, yielding a 95% CI of [-26.65 mg/dL, -20.92 mg/dL]. That is, our observed data are consistent with mean cholesterol truly being between 26.65 mg/dL lower and 20.92 mg/dL lower in men relative to women, after accounting for race. The accompanying two-sided p-value testing whether the difference in mean cholesterol between males and females (after accounting for race) is less than $10^{-3}$, so we again have evidence of an association between cholesterol and sex among elderly individuals, after controlling for race.

**d.** Our analysis suggests that race modifies the association between mean cholesterol level and sex. We find that mean cholesterol between races is 9.6 mg/dL lower for males than it is for females in our data. The associated standard error estimate is 3.78 mg/dL, which yields a 95% CI of [-17.01 mg/dL, -2.19 mg/dL]. We note that 0 is not contained in this interval. The accompanying two-sided p-value testing whether the difference in mean cholesterol across races is the same in both males and females is 0.011 < 0.05, so we have evidence that race modifies the association between mean cholesterol level and sex.

**(2)**

**Methods:** We use the same methods as in problem 1.

**a.** We find that the mean fibrinogen for Caucasian males in our data is 2.9 mg/dL lower than the mean fibrinogen level of the Caucasian females in our data. The associated standard error for this estimate is 2.68 mg/dL, yielding a 95% CI of [-8.15 mg/dL, 2.35 mg/dL]. We note that 0 is contained within this interval. That is, our observed data are consistent with mean fibrinogen truly being between 8.15 mg/dL lower and 2.35 mg/dL higher for elderly Caucasian males than for elderly Caucasian females. The accompanying two-sided p-value for a test of the true difference in means between these two groups being 0 is 0.279 > 0.05 so we do not have evidence of an association between fibrinogen and sex among elderly Caucasians.

**b.**

We find that the mean fibrinogen for Non-Caucasian males in our data is 15.7 mg/dL lower than the mean fibrinogen level of the Non-Caucasian females in our data. The associated standard error for this estimate is 7.30 mg/dL, yielding a 95% CI of [-30.0 mg/dL, -1.40 mg/dL]. We note that 0 is not contained within this interval. That is, our observed data are consistent with mean fibrinogen truly being between 30 mg/dL lower and 1.4 mg/dL lower for elderly Non-Caucasian males than for elderly Non-Caucasian females. The accompanying two-sided p-value for a test of the true difference in means between these two groups being 0 is 0.032 < 0.05 so we have evidence of an association between fibrinogen and sex among elderly Non-Caucasians.

**c.**

* *Importance weights:* Using the relative frequencies in our sample, we weight the Caucasian stratum by 0.8405 and the Non-Caucasian stratum by 0.1595. After adjustment for race, we estimate that mean fibrinogen for males is 4.94 mg/dL lower than mean fibrinogen for females. The associated standard error for this estimate is 2.53 mg/dL, yielding a 95% CI of [-9.91 mg/dL, 0.031 mg/dL]. That is, our observed data are consistent with mean fibrinogen truly being between 9.91 mg/dL lower and 0.031 mg/dL higher in men relative to women, after accounting for race. The accompanying two-sided p-value testing whether the difference in mean fibrinogen between males and females (after accounting for race) is 0.0514 > 0.05, so we do not have evidence of an association between fibrinogen and sex among elderly subjects, after controlling for race.
* *Efficiency weights:* Here, we attach a weight of 0.8812 to the Caucasians stratum and a weight of 0.1188 to the Non-Caucasian stratum. After adjustment for race, we estimate that mean fibrinogen for males is 4.42 mg/dL lower than mean fibrinogen for females. The associated standard error for this estimate is 2.52 mg/dL, yielding a 95% CI of [-9.35 mg/dL, 0.510 mg/dL]. We note that 0 is contained within this interval. That is, our observed data are consistent with mean fibrinogen truly being between 9.35 mg/dL lower and 0.510 mg/dL higher in men relative to women, after accounting for race. The accompanying two-sided p-value testing whether the difference in mean fibrinogen between males and females (after accounting for race) is 0.079 > 0.05, so we again do not have evidence of an association between fibrinogen and sex among elderly subjects, after controlling for race.

**d.** Our analysis does not suggest that race modifies the association between mean fibrinogen level and sex. We find that mean fibrinogen between races is 12.8 mg/dL higher for males than it is for females in our data. The associated standard error estimate is 7.78 mg/dL, which yields a 95% CI of [-2.44 mg/dL, 28.04 mg/dL]. We note that this interval contains 0. The accompanying two-sided p-value testing whether the difference in mean fibrinogen across races is the same in both males and females is 0.10 > 0.05, so we do not have evidence that race modifies the association between mean fibrinogen level and sex.

**(3)**

In our data, 47 subjects have missing cholesterol values (coded as “NA”). As a result, we exclude these observations from our analysis for the following questions.

**a.** Among the 4,953 individuals with non-missing cholesterol values, our best estimate of the sample standard deviation of cholesterol is 39.29 mg/dL.

**b.** If we assume that $ρ=0.40$, we estimate the standard deviation of the change in cholesterol measurements made after 3 years in the population will be 55.27 mg/dL.

**c.** Among groups have constant age and sex, we estimate a within-group standard deviation of cholesterol of 37.49 mg/dL.

**(4)**

**a.** The sample size that provides 80% power to detect the design alternative will be

$$N= \frac{2.801^{2}×(8×37.49^{2}×0.6)}{\left(0-10\right)^{2}}=529.30 ≈530$$

**b.** The sample size that provides 90% power to detect the design alternative will be

$$N= \frac{3.242^{2}×(8×37.49^{2}×0.6)}{\left(0-10\right)^{2}}=709.08 ≈710$$

**c.** If we had not adjusted for age and sex, the sample size necessary for 90% power would be

$$N= \frac{3.242^{2}×(8×39.29^{2}×0.6)}{\left(0-10\right)^{2}}=778.91 ≈779$$

**d.** If we had analyzed only the final cholesterol measurement adjusted for age and sex, the sample sizes necessary for us to achieve a fixed level of power would decrease. This is because we would no longer scale the first three terms in our numerator by $1-ρ=0.60$, so the numerator would be larger with the denominator held fixed.

**e.** If we had used an Analysis of Covariance model that adjusted for age, sex, and baseline cholesterol level, then our computed sample sizes would decrease. ANCOVA is a more efficient estimation method so it requires a smaller sample size to achieve the same power.

**(5)**

**a.**  We estimate that 39.57% of subjects on the control arm have cholesterol below 200 mg/dL at the end of the treatment period.

**b.** We estimate that 49.43% of subjects on the treatment arm have cholesterol below 200 mg/dL at the end of the treatment period.

**c.** The sample size that provides 90% power to detect the design alternative will be

$$N= \frac{3.242^{2}×2×\left(0.3957×\left(1-0.3957\right)+0.4943×(1-0.4943)\right)}{(0.3957-0.4943)^{2}}=1057.52≈1058 $$

**d.** A disadvantage of this study design compared to problem 4(b) is that here we have dichotomized our cholesterol measurement and thus have lost information about the distribution of cholesterol in our sample. This study design also requires a significantly larger sample size to provide 90% power than the design in 4(b), so it may be more costly to implement.