Biost 518
Applied Biostatistics II

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Lecture 12:
Correlated Response;
Weighted Regression

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Lecture Outline

- Dependent data within clusters
- Weighted regressions

Dependent Data Within Clusters

Dependent Data

- There are times when data can not be presumed to be totally independent
  - Sampling within families
  - Sampling within schools, hospitals
  - Repeated measurements on individuals taken at a single time
  - Longitudinal data: repeated measurements taken on individuals over time

Motivation for Longitudinal Data

- Three settings in which longitudinal studies are performed
  - Convenience of existing study population
  - Efficiency
    - Repeated measurements to decrease variability
    - Using subjects as own comparison
  - Scientific questions about effects that occur
    - over time, or
    - within subjects

Convenience

- Questions are truly cross-sectional
  - Multiple measurements made on each individual is easier than gathering new subjects
    - Natural variation within individuals provides additional information
  - E.g., Serum osmolality from Na, Glc, BUN
    - Interest is relationships between concurrent measurements

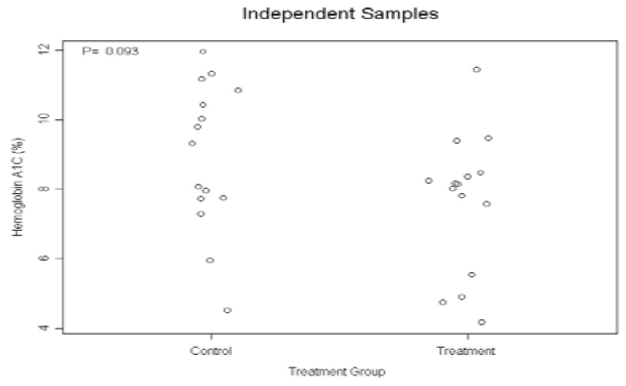
Efficiency

- Questions could be answered with cross-sectional study
- Primary comparison within subjects may have less variability
  - Allow detection of smaller effects
  - E.g., Adjusting for baseline measurements
  - E.g., Cross-over study of a new treatment

Example

- Percent glycosylated hemoglobin is used to monitor long term control in diabetes
  - Hemoglobin A1c
- Consider studies of two insulin delivery strategies
  - Independent groups
  - Cross-over design

Graph: Independent Samples

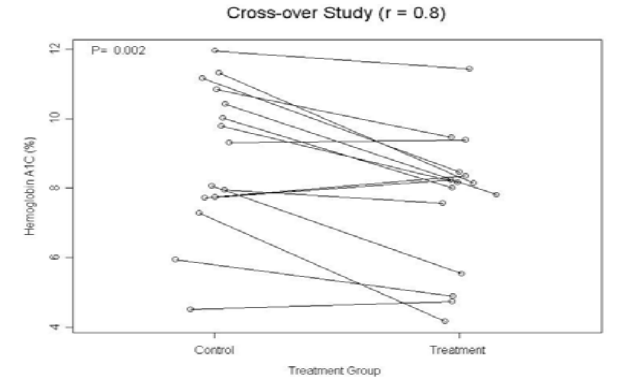


Inference: Independent Groups

- Large between-subject variability hampers our ability to detect differences
  - Between group SE is square root of sum of squared within group SEs
  - Within group SEs are proportional to within group standard deviation divided by the square root of n

$$se\left(\overline{X}-\overline{Y}\right)=\sqrt{\frac{\sigma_X^2}{n_X}+\frac{\sigma_Y^2}{n_Y}}$$

Graph: Cross-over Study



Inference: Cross-over Study

- High correlation between measurements taken on the same individual increases precision
  - The “random effect” of patient ID can be thought of as a precision variable

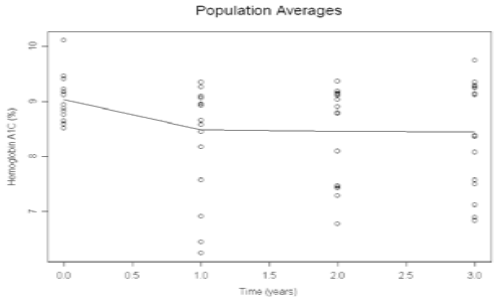
$$se\left(\overline{X}-\overline{Y}\right)=se\left(\overline{D}\right)=\sqrt{\frac{\sigma_X^2+\sigma_Y^2-2\rho\sigma_X\sigma_Y}{n}}$$

Longitudinal Questions

- Scientific questions about effects that occur over time
  - Studies to detect population time trends in response
    - E.g., rate (slope) of progression of retinopathy in population of diabetics over time
    - E.g., time to development of albuminuria

Example: "Marginal Effects"

- Time trends in group mean HbA1C
  - Note trends in mean and variability



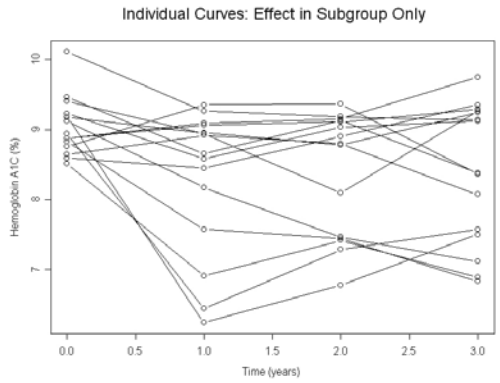
Within Subject Effects

- Trends in specific individuals might not look like trends in population means
  - Response over time may be restricted to subgroups of subjects
  - Response over time may be transient

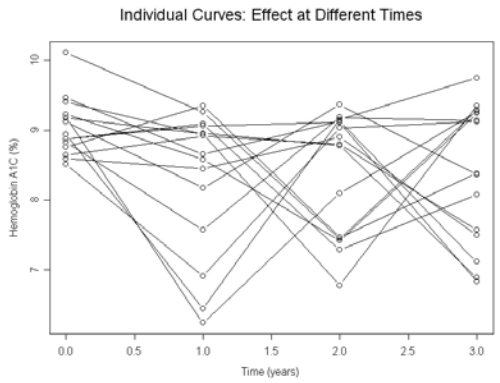
Longitudinal Scientific Questions

- Scientific questions about effects that occur within subjects
  - Studies to detect time trends or covariate effects in individual response
    - E.g., distribution of rates (slopes) of progression of retinopathy in population over time
    - E.g., effect of varying risk factors within individuals

Effect in Subgroup



Transient Effects



Choice of Measures of Outcome

- In order of importance
  - Scientific relevance
    - Including state of current knowledge
  - Plausibility of difference across groups
  - Statistical precision for analysis

Longitudinal Outcome Measures

- In longitudinal studies, each individual may have multiple measurements over time
  - Definition of individual response thus can be based on multiple measurements
    - Response at a fixed time
    - Responses at multiple fixed times
    - Average response over time (area under curve)
    - Rate of change in response (slope)
    - Time to attaining some level of response

Measures of Outcome

- "Marginal" or population effects
  - Difference or ratio of group means, geometric means, medians, proportion or odds above threshold, hazards
  - Pr (Y > X)
- "Within subject" effects
  - Mean, median difference
  - Mean, geometric mean, median ratio
  - Within subject odds ratio
  - Pr (Y > X)

Choice of Longitudinal Outcome

- Should reflect scientific relevance, plausibility of effect, precision
  - Final level of response may be more important than earlier effects
    - (But in the long run, we are all dead)
  - Summarizing response at multiple time points reflects population rather than individuals
  - Average response over time sensitive to transient effects
  - Differences in time to event may be clinically meaningless

Statistical Issues

- Repeated measurements on subjects require special analysis techniques
  - May have erroneous conclusion if fail to account for correlated observations
    - Point estimates may be biased for population parameters
      - Too much emphasis placed on some subjects
    - Confidence intervals will not be accurate representation of our true confidence
    - P values will be wrong

Statistical Approaches

- Three basic approaches to analyzing correlated data
  - Reduce measurements on each cluster to a single observation; analyze across clusters
  - Estimate correlation within clusters and adjust standard errors for population based models
    - GEE, marginal models
    - "Robust" variance estimates
  - Adjust estimates for "random effects"
    - "Mixed effects models": both fixed and random

Easiest Approach

- Reduce data for each individual to a single measurement
  - E.g., response at end of study, average response, rate of change
  - Analyses can then be based on standard methods for independent data
  - But:
    - Does not allow time-varying covariates
    - May not be most efficient statistically

Example: Beta-carotene Data

- Randomized clinical trial of beta-carotene supplementation on plasma levels of beta-carotene and vitamin E
  - Subjects randomized to 5 dose groups
  - Measurements at baseline, after 3 and 9 months of treatment, and 3 months after stopping treatment
  - Scientific question: How do plasma beta-carotene levels change over time within dose groups?
    - (effect modification between dose and time)

Example: Beta-carotene Data

- Reduce data to a single measurement on each subject
  - Difference between follow-up and baseline
    - Consider average of differences
    - No change corresponds to a difference of 0
  - Ratio between follow-up and baseline
    - Consider average of ratios
    - No change corresponds to a ratio of 1

Example: SEP data

- Somatosensory evoked potential measurements on healthy adults
  - Measurements of nerve conduction time
    - Four separate peaks for each leg of each subject
  - Reduce data to a single measurement
    - Consider only one peak on one leg
      - Which one?
    - Average measurements across peaks, legs
      - But will only generalize to similar averages
    - (Differences between peaks?)

Two Matched Samples

- Paired t test for means
- Sign test for median difference
- Wilcoxon signed rank test
- McNemar's test for difference in proportions or odds ratio
  - Equivalent to sign test

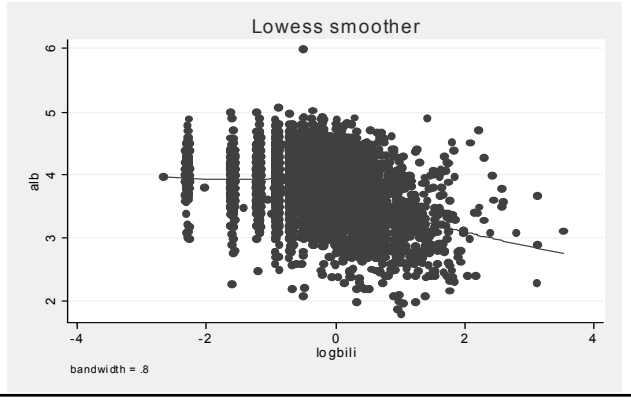
General Approach

- Regression models which allow correlated data within identified clusters
  - Allows time-varying covariates
  - Allows greater precision in some settings
    - Recall increased precision by adjusting for baseline as a predictor in RCT

Example: Bilirubin & Albumin

- How do albumin levels relate to bilirubin levels in PBC?
  - Randomized clinical trial
    - 265 subjects
    - Measurements made every 3 months
      - 9,068 measurements
  - Transform to log bilirubin for skewness

Lowess Smooth: Alb vs Logbili



Regression Results

```
. regress alb logbili, robust
Linear regression               Number of obs =    9068
                                F(  1,  9066) =  464.37
                                Prob > F      =  0.0000
                                R-squared     =  0.0729
                                Root MSE      =  .39451
------------------------------------------------------------------------------
             |               Robust
         alb |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     logbili |   -.171.   .0080   -21.55  0.000    -.187    -.156
       _cons |    3.82.   .0060   635.72  0.000     3.81     3.83
------------------------------------------------------------------------------
```


So far: Inferential Assumptions

- Linear regression
  - Independence
    - Independent observations
  - Variance
    - Equal variance or robust standard errors
  - Normally distributed estimates
    - Large sample size
      - If normally distributed data within groups, 2 observations are “large”

Now: Inferential Assumptions

- Linear regression
  - Independence
    - Independent observations ***between clusters***
  - Variance
    - Equal variance or robust standard errors
  - Normally distributed estimates
    - Large sample size
      - If normally distributed data within groups, 2 observations are “large”

Adjusting for Correlated Data

- True standard errors need to account for correlated data
  - Adjustment will depend on
    - Whether statistic involves sums or differences of correlated observations
    - Whether data are positively or negatively correlated

Effect of Correlated Data

- If statistic adds correlated observations
  - Positively correlated data leads to larger SE than independent data
- If statistic subtracts correlated observations
  - Positively correlated data leads to smaller SE than independent data

$$Var\left(Y_i + Y_j\right) = Var\left(Y_i\right) + Var\left(Y_j\right) + 2\rho\sqrt{Var\left(Y_i\right)Var\left(Y_j\right)}$$

$$Var\left(Y_i - Y_j\right) = Var\left(Y_i\right) + Var\left(Y_j\right) - 2\rho\sqrt{Var\left(Y_i\right)Var\left(Y_j\right)}$$

Example: "Repeated Measures"

- Subjects in a group have multiple measurements
  - E.g., Average blood pressure by sex
    - Repeated measurements on subjects who are always in the same group
      - Likely tend to be positively correlated
    - Repeated measurements added to get mean
    - Failure to account for correlated data will tend to underestimate true SE
      - "Anti-conservative" inference: P values too low, CI too narrow

Example: "Crossover"

- Subjects contribute information to different groups
  - E.g., RCT with each subject on placebo and active treatment
    - Positively correlated measurements subtracted when computing difference in group means
    - Failure to account for correlated data will tend to overestimate true SE
      - "Conservative" inference
      - P values too high, CI too wide

Example: Combination

- Regression sometimes adds, sometimes subtracts correlated observations
  - Depends on whether predictors for repeated measures are larger or smaller than mean

$$\text{Model} : \quad E\left[Y_i \mid X_i\right] = \beta_0 + \beta_1 \times X_i$$

$$\text{Estimate} : \quad \hat{\beta}_1 = \frac{\sum_{i=1}^{n} Y_i\left(X_i - \overline{X}\right)}{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}$$

- Failure to account for correlated data can lead to either conservative or anti-conservative inference

Adjusting for Correlation

- Huber – White "sandwich" estimator
  - "Robust standard error estimates"
    - Adjust for unequal variances
    - Adjust for correlation within identified clusters
  - Does not change the regression parameter estimates

Stata: Adjusting for Correlation

- With any regression command use option
  - "…, cluster(*varname*)"
  - The cluster variable is usually nominal
  - Can also specify "robust", but Stata assumes that robust SE should be used whenever clusters are identified

Presuming Independence

```
. regress alb logbili, robust
Linear regression            Number of obs =    9068
                             F(  1,   9066) =  464.37
                             Prob > F      =  0.0000
                             R-squared     =  0.0729
                             Root MSE      =  .39451
        |       Robust
    alb |  Coef    SE       t    P>|t|    [95% C I]
logbili | -.171 .0080  -21.55 0.000 -.187   -.156
  _cons |  3.82 .0060  635.72 0.000  3.81    3.83
```


Adjusting for Clusters

```
. regress alb logbili, robust cluster(ptid)
Linear regression            Number of obs =    9068
                             F(  1,    264) =   38.29
                             Prob > F      =  0.0000
                             R-squared     =  0.0729
No. clusters (ptid) = 265    Root MSE      =  .39451
        |       Robust
    alb |  Coef    SE       t    P>|t|    [95% C I]
logbili | -.171 .0277   -6.19 0.000 -.226   -.117
  _cons |  3.82 .0235  162.19 0.000  3.77    3.86
```

Important Caveat

- This approach does not alter the regression parameter estimates
  - By default, each observation is weighted equally
    - Science:
      - Perhaps each subject should be weighted equally
    - Statistics:
      - Perhaps more precise measurements should be weighted more heavily

Regression on Binary Predictors

Means: Linear Regression

- Classical regression
  - t test which presumes equal variance
- Robust standard errors
  - t test which allows unequal variance (approx)
- Robust standard errors with identified cluster variable
  - paired t test (approx)

Example: Classical LR

- In PBC placebo patients, a comparison of mean bilirubin levels between patients with and without edema
  - Standard variance estimates: Compare
    - "`ttest bili, by edema`", and
    - "`regress bili edema`"

Standard Variance Estimates

```
. ttest bili if treatmnt==2, by(edema)
Group| Obs     Mean     StErr.   StDev.      [95% CI]
   0 | 137    3.018     .387     4.535    2.252    3.784
   1 |  16    9.25     1.941     7.764    5.113   13.387
diff |       -6.232    1.308             -8.816   -3.647
Ha: diff < 0     Ha: diff ~= 0              Ha: diff > 0
t =  -4.764      t =  -4.764                t =  -4.764
P<t= 0.0000      P> |t|= 0.0000           P > t = 1.000

. regress bili edema if treatmnt==2
      |             Robust
 bili | Coef.   StErr.      t    P>|t|          [95% CI]
edema | 6.232   1.308      4.76   0.000      3.647    8.816
_cons | 3.018    .423      7.14   0.000      2.182    3.854
```


Correspondence

- Estimated intercept: the group 0 sample mean
  - Standard errors differ, because standard regression assumes equal variance between the groups and uses a pooled estimate of variance
  - CI will therefore also be different
- Estimated slope: the difference between group sample means
  - Standard error for slope is the standard error for the difference in means
  - CI, t statistic, and P value are exactly the same

Example: LR w/ Robust SE

- In PBC placebo patients, a comparison of mean bilirubin levels between patients with and without edema
  - Robust variance estimates: Compare
    - "`ttest bili, by(edema) unequal`", and
    - "`regress bili edema, robust`"

Robust Variance Estimates

```
. ttest bili if treatmnt==2, by(edema) unequal
Group| Obs     Mean     StErr.   StDev.      [95% CI]
   0 | 137    3.018     .387     4.535    2.252    3.784
   1 |  16    9.25     1.941     7.764    5.113   13.387
diff |       -6.232    1.979             -10.423   -2.040
Ha: diff < 0     Ha: diff ~= 0              Ha: diff > 0
t =  -3.1484     t =  -3.1484               t =  -3.1484
P < t = 0.0031   P > |t| = 0.0061           P > t = 0.9969

. regress bili edema if treatmnt==2, robust
      |             Robust
 bili | Coef.   StErr.      t    P>|t|          [95% CI]
edema | 6.232   1.931      3.23   0.002      2.416    10.048
_cons | 3.018    .389      7.77   0.000      2.250     3.786
```

Correspondence

- Estimated intercept: the group 0 sample mean
  - Standard errors agree more here
  - CI differs a bit more, because of difference in degrees of freedom: t test used 16, regression used 151
- Estimated slope: the difference between group sample means
  - Standard error for slope is approximately the standard error for the difference in means
  - CI, t statistic, and P value are about the same but again are influenced by degrees of freedom

Effect of Heteroscedasticity

- Note the problem with using standard linear regression in the presence of unequal variances
  - Distribution of predictor (edema) was skewed toward the group with the lower variance
    - Sample size largest in group with smaller variance
  - We thus expect standard error estimates from classical regression to be too low
    - Anti-conservative inference

Example: Classical LR (Wrong)

- In beta carotene datasets, a comparison of mean vitamin E levels at baseline and after 3 months of treatment (ignoring dose for the purpose of this illustration)
  - Standard variance estimates: Compare
    - "`ttest vite0=vite1`", and
    - "`regress vite time`" on reshaped data

Classical LR on All Data

```
ttest vite0=vite1
```

Varbl	Obs	Mean	StErr.	StDev.	[95% CI]	
vite0	45	8.025	.191	1.280	7.640	8.409
vite1	45	8.859	.135	.907	8.586	9.131
diff	45	-.834	.133	.894	-1.103	-.566

```
Ha: mn(diff) < 0    Ha: mn(diff) ~= 0    Ha: mn(diff) > 0
t =  -6.2598        t =  -6.2598         t =  -6.2598
P < t= 0.0000       P > |t|= 0.0000      P > t= 1.0000
```

```
regress vite time if time==0 | time==1
                              Number of observations =    91
```

vite	Coef.	StErr.	t	P>|t|	[95% CI]	
time	.799	.234	3.41	0.001	.334	1.264
_cons	8.060	.165	48.98	0.000	7.733	8.387

Correspondence

- In this example, no correspondence between the two methods of analysis
  - Analyses was performed on different datasets
    - Case 40 did not have measurements at 3 months
    - The paired t test did not use any part of that subject's data
    - The regression used the data at baseline
    - (We can restrict the regression to use the same data)

Ex: Classical LR on Same Data

```
. ttest vite0=vite1
Varbl | Obs     Mean     StErr.    StDev.          [95% CI]
vite0 |  45    8.025     .191     1.280     7.640     8.409
vite1 |  45    8.859     .135      .907     8.586     9.131
diff  |  45    -.834     .133      .894    -1.103     -.566
Ha: mn(diff) < 0    Ha: mn(diff) ~= 0    Ha: mn(diff) > 0
t =  -6.2598        t =  -6.2598         t =  -6.2598
P < t= 0.0000       P > |t|= 0.0000      P > t= 1.0000

. regress vite time if (time==0 | time==1) & ptid!=40
                            Number of observations =    90
 vite | Coef.   StErr.      t    P>|t|     [95% CI]
 time |  .834    .234     3.57   0.001     .370  1.299
_cons | 8.025    .165    48.54   0.000    7.696  8.353
```


Correspondence

  - Estimated intercept: the group 0 sample mean
    - Standard errors differ, because standard regression assumes equal variance between the groups and uses a pooled estimate of variance
    - CI will therefore also be different
  - Estimated slope: the difference between group sample means
    - The mean of a difference is difference of means
    - Inference is wrong because it does not account for dependent observations

Ex: LR w/ Robust SE (Wrong)

- In beta carotene datasets, a comparison of mean vitamin E levels at baseline and after 3 months of treatment (ignoring dose for the purpose of this illustration)
  - Robust variance estimates: Compare
    - "`ttest vite0=vite1`", and
    - "`regress vite time, robust`" on reshaped data

Ex: Robust SE; No Clusters

```
. ttest vite0=vite1
Varbl | Obs     Mean     StErr.    StDev.          [95% CI]
vite0 |  45    8.025     .191     1.280     7.640     8.409
vite1 |  45    8.859     .135      .907     8.586     9.131
diff  |  45    -.834     .133      .894    -1.103     -.566
Ha: mn(diff) < 0    Ha: mn(diff) ~= 0    Ha: mn(diff) > 0
t =  -6.2598        t =  -6.2598         t =  -6.2598
P < t= 0.0000       P > |t|= 0.0000      P > t= 1.0000

. regress vite time if (time==0 | time==1) & ptid!=40,
      robust
                        Number of observations =     90
 vite | Coef.   StErr.       t    P>|t|      [95% CI]
 time |  .834    .234      3.57   0.001      .370  1.299
_cons | 8.025    .191     42.07   0.000     7.646  8.404
```


Correspondence

- Estimated intercept: the group 0 sample mean
  - Standard errors agree more here
  - CI differs a bit more, because of difference in degrees of freedom: t test used 44, regression used 88
- Estimated slope: the difference between group sample means
  - Inference is still wrong because it does not account for dependent observations
    - When no clusters specified, robust SE only accounted for possibility of unequal variances, not dependence of data

Example: LR with Clusters

- In beta carotene datasets, a comparison of mean vitamin E levels at baseline and after 3 months of treatment (ignoring dose for the purpose of this illustration)
  - Robust variance estimates in clusters: Compare
    - "`ttest vite0=vite1`", and
    - "`regress vite time, robust cluster(id)`" on reshaped data

Ex: LR w/ Robust SE; Clusters

```
. ttest vite0=vite1
Varbl | Obs     Mean     StErr.    StDev.          [95% CI]
vite0 |  45    8.025     .191     1.280     7.640     8.409
vite1 |  45    8.859     .135      .907     8.586     9.131
diff  |  45    -.834     .133      .894    -1.103     -.566
Ha: mn(diff) < 0    Ha: mn(diff) ~= 0    Ha: mn(diff) > 0
t =  -6.2598        t =  -6.2598         t =  -6.2598
P < t= 0.0000       P > |t|= 0.0000      P > t= 1.0000

. regress vite time if (time==0 | time==1) & ptid!=40,
      robust cluster(ptid)
                        Number of observations =     90
 vite | Coef.   StErr.       t    P>|t|      [95% CI]
 time |  .834    .134      6.22   0.000      .564  1.104
_cons | 8.025    .192     41.83   0.000     7.638  8.411
```

Correspondence

- Estimated intercept: the group 0 sample mean
  - Standard errors agree more here
  - CI differs a bit more, due to degrees of freedom: t test used 44, regression used 88
- Estimated slope: the difference between group sample means
  - Standard error for slope is approximately the standard error for the mean difference
  - CI, t statistic, and P value are about the same but again are influenced by degrees of freedom

Effect of Correlated Data

- Note the problem when clustered data not identified
  - The observations at baseline and 3 months were positively correlated
  - Within clusters, the predictor of interest (time) differed
    - Each cluster had both a baseline and a 3 month measurement
  - SE from classical regression to be too high
    - Conservative inference (loss of power)

Ex: Clustered Analysis; All Data

```
. ttest vite0=vite1
Varbl | Obs     Mean     StErr.    StDev.          [95% CI]
vite0 |  45    8.025     .191     1.280     7.640     8.409
vite1 |  45    8.859     .135      .907     8.586     9.131
diff  |  45    -.834     .133      .894    -1.103     -.566
Ha: mn(diff) < 0   Ha: mn(diff) ~= 0   Ha: mn(diff) > 0
t =  -6.2598       t =  -6.2598        t =  -6.2598
P < t= 0.0000      P > |t|= 0.0000     P > t= 1.0000

. regress vite time if (time==0 | time==1), robust
      cluster(ptid)
                        Number of observations =    91
 vite | Coef.   StErr.      t    P>|t|     [95% CI]
 time |  .798   .136      5.88   0.000      .525  1.072
_cons | 8.060   .191     42.20   0.000     7.676  8.445
```


Correspondence

- The patient with missing data at 3 months now influences the baseline mean
  - Thus we no longer have correspondences with the paired t test, which cannot use partial information on a subject
- Relative appropriateness of approaches depends on whether
  - "Missing at random": Subjects missing data are similar to other subjects having the same values of the modeled variables
  - Interest in population differences or within subject differences

Aside: Handling Missing Data

- Two basic approaches
  - Omit cases with missing data
  - Impute missing data
    - Predict what missing data would have been

Aside: Omitting Cases

- If cases missing data are in some way different in (unknown) response values
  - Unbiased estimates of population that is not prone to missing data
    - Condition on such a population
  - Biased if cases missing data are different
- If cases missing data are "missing at random" based on modeled data
  - May lose power (relative to imputing)
    - Depends on amount of information derived from imputing

Aside: Imputing Data

- Predict missing data from a regression model or matching scheme
  - Can use all available data, including variables not otherwise included in the data analysis
  - Accurate to the extent you have the right prediction model
- Account for variability
  - Predict individual observation (mean + noise)
  - Multiple imputation for better SE

Correlated Data Analysis

- The robust variance estimates modify the standard errors, not the estimated slope
  - Estimated slopes are the same as if every subject were independent
    - Each subject is not weighted equally if the sample sizes per subject are not the same
  - This is OK for testing associations, because the standard errors will estimate the sampling variability correctly
    - But for esthetic reasons, we might prefer estimates which weight each subject appropriately

Correlated Data Analysis

- Regression parameter estimates will generally be "population" rather than "within individual" effects
  - "Population effects"
    - Estimates refer to comparing different subjects who have different predictor values
    - "Marginal models", e.g., "Generalized Estimating Equations (GEE)
  - "Within individual effects"
    - Estimates refer to comparing measurements on the same subject when he/she has different predictor values
    - "Random effects" in "Mixed models"

Weighted Regression

Weighted Analyses

- When sampling of individuals does not reflect their importance in the population, we should re-weight observation
  - Biost 540 will address issues to choice of models with dependent outcome data more fully
    - In this class, we will only consider
      - How to ensure accurate inference, and
      - How to obtain estimates which weight clusters equally in a simple manner

Choice of Weights

- The importance we should give each "cluster" (individual) should depend upon
  - the scientific question
  - the regression modeling of the predictors
  - the distribution of predictors across individuals
  - the reasons that we might have fewer observations for some subjects (e.g., nonignorable missingness?)

Weighted Regression

- Allows more emphasis to be placed on some observations than others
  - Stata classification:
    - "Frequency weights"
    - "Analytic weights"
    - "Probability weights"
  - In fact, all these weighting schemes actually use the same mathematical techniques
    - Vary in the way the results are presented

Weighted Regression in Stata

- Every regression command in Stata can take weights
  - regress yvar xvar [wttype=expr] ...
    - could substitute "logistic", "poisson", "stcox" for "regress"
    - square brackets [ ] are necessary
    - wttype is one of "fweight", "pweight", or (only for "regress") "aweight"
    - expr is an expression typically involving some variable

Frequency Weights

- Adjust for duplicate cases that have the exact same measurements: Weights proportional to the frequency
  - Sometimes for reasons of storage efficiency, we tabulate the number of cases having the same response and predictors
  - We then have an additional variable that represents the frequency with which each observation occurs in the sample

Ex: Frequency Weights

- Frequency weights have greatest use in logistic regression
  - For each combination of predictors have
    - One case counting number of subjects with response = 1
    - One case counting number of subjects with response = 0
  - Often taken from registry or census data

Analytic Weights (Lin Reg Only)

- Adjust for cases which were measured with greater precision than others
  - Weights proportional to the inverse of the variance
    - E.g., the response is an average of several measurements
      - More measurements used in average = more precision
      - Number of measurements may not be the same for each case
      - Weights are the number of measurements used to compute the response

Uses of Analytic Weights

- Adjust for heteroscedasticity
  - Analytic weights are inversely proportional to the variance of the individual case
    - We would need to know the variance in each predictor group
    - Then weight individuals according the value of their predictors
      - Can be quite difficult with multiple predictors
- (Sandwich estimator is much to be preferred)

Probability Weights

- Adjust "oversampling" or "undersampling" some elements of the population
  - Sometimes some segments of the population have not been sampled in direct proportion to their importance in the population
    - Weights proportional to the inverse of the sampling probability
    - E.g., multiple measurements on same individual
      - Each individual is equally important in the population
      - Each individual may not have the same number of measurements in the population

Use of Probability Weights

- It is the probability weights that we are most interested in for this course
  - We can use them to adjust regression estimates in order to reflect equal emphasis placed on each individual
  - We compute the "empirical probability" of sampling each observation as proportional to the count of observations for each "cluster"

Ex: Salary by Year 1990 - 1995

- How does geometric mean salary differ by year between 1990 and 1995
  - Correlated observations on faculty
  - Some individuals were hired between 1991 and 1995
  - Fewer observations for those subjects

Ex: Classical LR (Wrong)

```
. drop if year <90
(11239 observations deleted)
. g logslry= log(salary)
. regress logslry year
```

Source	SS	df	MS	Number of obs =	8553
Model	17.4	1	17.4	F(1, 8551) =	196.59
Residual	757.3	8551	.089	Prob > F =	0.0000
Total	774.7	8552	.091	R-squared =	0.0225
				Adj R-squared =	0.0224
				Root MSE =	.29759

logslry	Coef	SE	t	P>|t|	[95% C I]	
year	.0266	.00190	14.02	0.000	.023	.030
_cons	6.189	.17565	35.24	0.000	5.84	6.53

Ex: Only Robust SE (Wrong)

```
. regress logslry year, robust
```

Linear regression		Number of obs =	8553
		F(1, 8551) =	198.22
		Prob > F =	0.0000
		R-squared =	0.0225
		Root MSE =	.29759

logslry	Coef	Robust SE	t	P>|t|	[95% C I]	
year	.0266	.00189	14.08	0.000	.023	.030
_cons	6.189	.17483	35.40	0.000	5.85	6.53

Ex: Cluster(ID) (Wrong Weights)

```
. regress logslry year, robust cluster(id)
```

Linear regression		Number of obs =	8553
		F(1, 1596) =	630.86
		Prob > F =	0.0000
		R-squared =	0.0225
Nbr of clusters (id)= 1597		Root MSE =	.29759

logslry	Coef	Robust SE	t	P>|t|	[95% C I]	
year	.0266	.0011	25.12	0.000	.025	.0287
_cons	6.189	.0989	62.57	0.000	6.00	6.383

Comments

- Identifying clusters made inference more precise
  - Estimates were the same
  - Positively correlated observations within clusters
  - Predictor (year) varied within clusters
- But weighting counted some individuals more than others
  - We might want to weight individuals equally

Ex: Cluster(ID); Weighted

```
. egen cnt= count(id), by(id)
. regress logslry year [pw=1/cnt], robust
  cluster(id)
(sum of wgt is   1.5970e+03)
Linear regression              Number of obs =    8553
                               F(  1,  1596) =   55.40
                               Prob > F      =  0.0000
                               R-squared     =  0.0077
Nbr of clusters (id)= 1597     Root MSE      =  .30503
        |       Robust
logslry |  Coef    SE     t     P>|t|     [95% C I]
   year | .0157 .0021  7.44   0.000   .0116   .0198
  _cons | 7.179 .1946 36.88   0.000   6.797   7.561
```


Comments

- This weighted individuals equally
- However, because the individuals with less data were newer hires, there is some bias
  - Older hires apply to all years, newer hires to recent years
  - We would really rather have a model that considered the difference in salaries in order to get something closer to the within individual

Comments

- This weighted individuals equally
- Because the individuals with less data were newer hires, there is some bias
  - Older hires apply to all years, newer hires to recent years
  - Population effects are probably less of interest
    - Consider within individual effects by adjusting for year 95 data

Comments

```
. egen lslry95=max(logslry), by(id)
. regress logslry year lslry95 [pw=1/cnt] if year
  <95, robust cluster(id)
(sum of wgt is   1.2309e+03)
Linear regression               Number of obs =    6956
                                F(  2,  1522) =15908.31
                                Prob > F      =  0.0000
                                R-squared     =  0.9433
Nbr of clusters (id)= 1523      Root MSE      =  .07172
             |               Robust
logslry |  Coef    SE      t    P>|t|   [95% C I]
   year | .03661 .00057  64.16 0.000 .0355 .0377
lslry95 | .98917 .00595 166.17 0.000 .9775 1.001
  _cons | -3.385 .07404 -45.72 0.000 -3.53 -3.24
```


Comments

- Averaging over the past five years, but counting each faculty member equally:
  - Faculty have averaged 3.7% raises (95% CI 3.6% to 3.8%) over the past five years
  - Highly statistically significant (P < 0.0005)
- (Note that we had to decide whether we should weight years equally or faculty equally)