

Biost 517
Applied Biostatistics I

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Lecture 7:
Bivariate Descriptive Statistics

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Lecture Outline

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- Review of univariate descriptive statistics
- Purpose of bivariate descriptive statistics
- Stratified univariate descriptives
- Graphical
 - Stratified histograms, densities, boxplots
 - Scatterplots, least squares lines, smooths
- Correlation

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**Review of Univariate
 Descriptive Statistics**

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Univariate Descriptive Statistics

		Binary	Unordered	Ordered		
			Nominal	Categ	Quant	Cens
Entire Distnution	Frequency	OK	OK	OK	OK	
	Cum Freq	boring		OK	OK	KM
	Mode	boring	Sample	Sample	Density	
	Min / Max	boring		boring	OK	
Dicho- tomize	Proportion (or Odds)	OK	OK	OK	OK	KM
Quant- iles	Quantiles (25th, Mdn, 75th)	boring		OK	OK	KM
Means	Arithmetic	(Prop)		***	OK	(?KM)
	Geometric				OK	(?KM)
	Harmonic				OK	(?KM)
	Std Dev	boring			OK	(?KM)
	Skew, Kurt	boring			OK	(?KM)

Purpose of Bivariate Descriptive Statistics

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Purpose of Bivariate Description

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- Characterize the relationship between two variables
 - Detecting errors in data collection or data entry
 - Characterizing materials and methods
 - Assessing validity of assumptions
 - Basis for some estimates of association (inference)
 - Hypothesis generation (exploration/inference)

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Detecting Errors

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- Sometimes data measurements are not unusual univariately, but are unusual in combination
 - E.g., 6 foot tall 3 year olds
 - E.g., pregnant males
- Patterns may exist in missing data
 - Minorities may be more likely to be missing data on some medical procedures

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Materials and Methods

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- Describe patterns of sampling
 - E.g., minorities may tend to be younger
 - E.g., older subjects may tend to be women
 - E.g., smokers may tend to drink alcohol more

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Validity of Assumptions

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- Scientific
 - Confounding (ultimately involves 3 variables)
 - Effect modification (involves 3 variables)

- Statistical
 - E.g., assumptions about within group variance
 - E.g., assumptions about linearity of trends
 - E.g., influence of “outliers”

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Evidence of Associations

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- Two variables are said to be associated if the distribution of one variable differs across groups defined by the other variable

- E.g., if interested in determining whether sex and blood pressure are associated, see if
 - distribution of blood pressure differs between men and women, OR
 - proportion of men varies across groups defined by blood pressure measurements

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Quantify Associations

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- Describe “dose-response”

- How the effect differs across groups having ever larger differences in the grouping variable
 - E.g., linear response
 - E.g., S-shaped curves
 - E.g., U-shaped trends

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Hypothesis Generation

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- Examine sample to detect associations not previously considered

- Any such associations suggested by the data should be confirmed in an independent sample

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Stratified Univariate Descriptive Statistics

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Stratified Univariate Descriptives

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- Strata defined by one variable
- Continuous variables must be divided into categories
- Methods of dividing into categories
 - Scientific basis: Intervals with scientific meaning
 - Statistical basis: Intervals with equal sample sizes
 - Provides similar precision in each interval
 - But generally such intervals are not evenly spaced, thus detecting linear trends are difficult

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Examining Stratified Descriptives

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- Errors in data:
 - Unusual range by strata
- Materials and methods:
 - Describe central tendency, range by strata
- Validity of assumptions:
 - E.g., missing data by strata
 - E.g., equal variances across strata
 - E.g., linear trends in central tendency
- Evidence of associations:
 - E.g., difference in means, medians across strata

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Choice of Stratified Descriptives

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- By purpose of descriptives
 - Ability to assess errors
 - Ability to describe sample
 - Ability to examine validity of assumptions
 - Scientific relevance as estimates of association
- By type of data
 - Unordered versus ordered
 - Continuous versus discrete
 - Uncensored versus censored

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FEV Ex: Categorizing Age

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- Stata: Create variable containing age categories in two year intervals
 - g agetg = age
 - recode agetg 3/4=3 5/6=5 7/8=7 9/10=9 11/12=11 13/14=13 15/16=15 17/18=17 19/20=19
 - Alternative approach using arithmetic
 - g agetg = int ((age-1) / 2) * 2 + 1

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FEV Ex: Stata Commands

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- Stata: Create table of stratified statistics
 - by agetg: tabstat height, stat(n mean sd min p25 p50 p75 max) col(stat) format

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FEV Ex: Height by Age Groups

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<u>agetg</u>	<u>N</u>	<u>mean</u>	<u>sd</u>	<u>min</u>	<u>p25</u>	<u>p50</u>	<u>p75</u>	<u>max</u>
3	11.0	48.8	1.8	46.0	48.0	48.0	50.0	52.0
5	65.0	52.5	2.4	46.5	51.0	52.5	54.0	58.0
7	139.0	57.1	3.3	47.0	54.5	57.0	59.5	67.5
9	175.0	61.5	3.3	52.5	59.0	61.0	64.0	70.0
11	147.0	64.7	3.3	57.0	62.0	64.5	67.0	72.0
13	68.0	66.7	3.5	61.0	63.8	67.0	69.0	74.0
15	32.0	66.8	3.4	60.0	64.0	66.8	69.3	73.5
17	14.0	67.7	3.6	60.0	66.0	68.5	70.0	73.0
19	3.0	67.8	3.6	65.5	65.5	66.0	72.0	72.0
Total	654.0	61.1	5.7	46.0	57.0	61.5	65.5	74.0

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FEV Ex: Findings

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- Mean, median height within age strata increases by about 4 inches every two years up until about age 12 then levels off
- Standard deviation of height within age strata much less than standard deviation of entire sample
 - An indication that age predicts height
- Standard deviation of height within age strata increases as the mean increases

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Mean-Variance Relationships

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- We often see the variance differ systematically according to group means

- Differential diagnosis
 - Precision of measurement as a percentage
 - Variability in rates, but measurement of total
 - E.g., different growth per year, height at age 10
 - Confounding
 - E.g., more older kids smoke, which stunts growth?
 - Effect modification
 - E.g, young boys and girls tend to be same height, older boys taller than older girls

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FEV Ex: Age Quantiles

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- Stata: Find age quantiles
 - centile age,c(12 25 37 50 62 75 87)

Variable	Obs	Pctile	Centile	[95% Conf. Interval]	
age	654	12	7.0	6.0	7.0
		25	8.0	8.0	8.0
		37	9.0	9.0	9.0
		50	10.0	9.0	10.0
		62	11.0	10.0	11.0
		75	12.0	11.0	12.0
		87	13.0	13.0	14.0

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FEV Ex: Categories by Quantiles

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- Stata: Create variable of age categories
 - g agectg=age
 - recode agectg min/7=1 8=2 9=3 10=4 11=5 12=6 13=7 14/max=8
 - tabstat fev, stat(n mean sd min p25 p50 p75 max) col(stat) format by(agectg)

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FEV Ex: Height by Age Octiles

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agectg	N	mean	sd	min	p25	p50	p75	max
1	130.0	53.4	3.1	46.0	51.0	53.0	55.5	62.5
2	85.0	58.3	3.2	52.0	56.5	58.5	60.0	67.5
3	94.0	60.6	2.9	53.0	58.5	60.3	62.5	69.0
4	81.0	62.5	3.4	52.5	60.0	62.0	65.0	70.0
5	90.0	64.5	3.2	58.0	62.0	64.5	67.0	72.0
6	57.0	65.2	3.5	57.0	63.0	64.5	68.0	72.0
7	43.0	66.2	3.6	61.0	63.0	66.5	68.5	74.0
8	74.0	67.2	3.4	60.0	64.5	67.0	70.0	73.5
Total	654.0	61.1	5.7	46.0	57.0	61.5	65.5	74.0

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FEV Ex: Findings with Quantiles

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- We were less able to pick out the regions of linearity
 - The lowest octile covered several years, the next few octiles were each 1 year in width

- We were not able to pick out the mean variance relationship
 - The first octile was not as homogeneous with respect to age, and height varied with age within that stratum

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Crosstabulation of Categories

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- tabulate smoker female, cell column row

smoker	female		Total
	0	1	
	310	279	589
	52.63	47.37	100.00
	92.26	87.74	90.06
0	47.40	42.66	90.06
	26	39	65
	40.00	60.00	100.00
	7.74	12.26	9.94
1	3.98	5.96	9.94
Total	336	318	654
	51.38	48.62	100.00
	100.00	100.00	100.00
	51.38	48.62	100.00

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Graphical Bivariate Descriptive Statistics

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Stratified Univariate Graphs

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- Divide sample into strata based on one variable

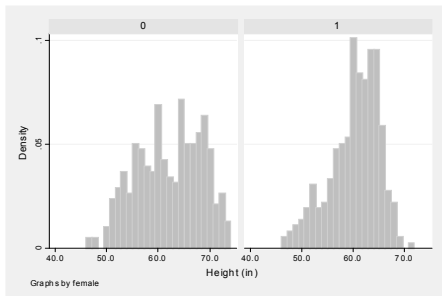
- Display for each stratum
 - histograms
 - densities
 - boxplots

- For greatest comparability, the same axes should be used in all plots

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Ex: Stratified Histograms

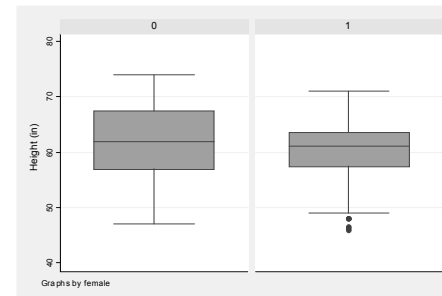
- FEV data: Histograms of height by sex
 - `hist height, by(female) xtitle("Height (in)")`
 - Note use of option `xtitle()` to label x axis



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Ex: Stratified Boxplots

- FEV data: Boxplots of height by sex
 - `graph box height, by(female) ytitle("Height (in)")`
 - Note use of option `ytitle()` to label x axis



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Ex: Stratified Univariate Stats

- FEV data: Univariate description of height by sex
 - `tabstat height, by(female) stat(n mean sd min p25 p50 p75 max) col(stat) format`

female	N	mean	sd	min	p25	p50	p75	max
0	336.0	62.0	6.3	47.0	57.0	62.0	67.5	74.0
1	318.0	60.2	4.8	46.0	57.5	61.0	63.5	71.0
Total	654.0	61.1	5.7	46.0	57.0	61.5	65.5	74.0

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Scatterplots

- A graph of Y versus X
 - Most useful for two continuous variables
- Look for
 - Outliers
 - Trends in location across groups
 - First order trends (linear)
 - Second order trends (curves, U-shape, S-shape)
 - Trends in within group spread of data
 - (Looking at range)

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Stata: Scatterplot

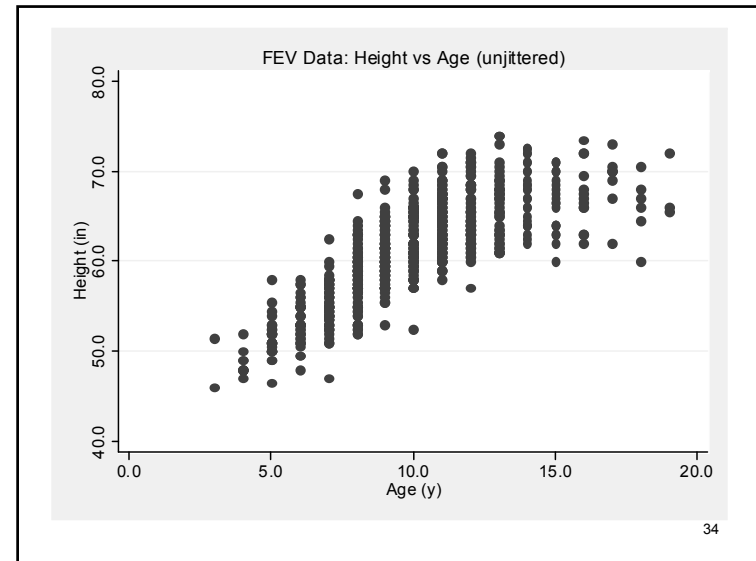
.....

- Stata commands
 - “scatter yvar xvar, [options]”
- Example: Height vs Age in FEV Data

```
scatter height age, xtitle("Age (y)")
t1("FEV Data: Height vs Age (unjittered)")
ytitle("Height (in)")
```

- Note use of options xtitle(), ytitle(), t1() to label x-axis, y-axis, and main title for plot

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Ex: Interpretation

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- No outliers
- Tends to increased height for older ages
 - First order trend is upward
- Hint of curvilinear relationship
 - Height levels off at highest ages
- Suggestion of increasing spread with increased height
 - Must be careful when judging variability from range
 - Need to compare range of equal numbers of data in area with equal slopes

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Jittered Scatterplots

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- If variables are discretely measured, jittering can be helpful
- “jittering”: adding a little noise to the data to break ties
- I tend to try to jitter to allow visualization of all points, but still try to keep discrete levels separate
 - I use a spread of about 40% the separation between categories

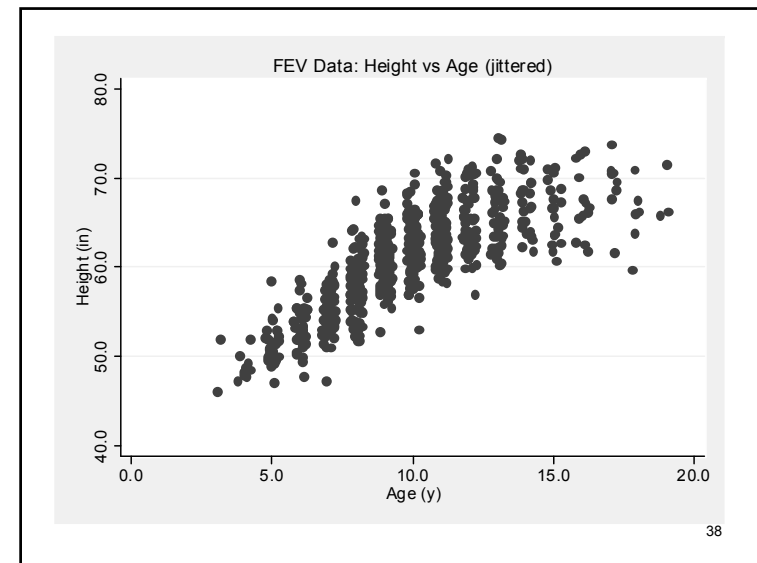
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Stata: Jittering

- Stata commands
 - “scatter yvar xvar, jitter(n)”
 - n is the percent of the plot width
 - Just guess a number and check
- Example: Height vs Age in FEV Data


```
scatter height age, xtitle("Age (y)")
      t1("FEV Data: Height vs Age (jittered)")
      ytitle("Height (in)") jitter(3)
```

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Variance Within Groups

- On a scatterplot, our eye sees range of data within groups
- We usually want to judge variance
 - Especially how variance might differ with X
- Converting range to variance
 - Consider spread in two regions far apart
 - Need sample sizes approximately equal
 - Need slopes approximately equal
 - Spread will be greater where slope is further from 0

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Superimposed Curves

- It is often helpful to place curves over a scatterplot to help see trends in the data
 - Theoretical relationship
 - If theory prescribes a supposed relationship
 - Least squares line of within group means
 - “Best fitting” line to means, but has to be a line
 - Smooths of measure of location within groups
 - Curve representing approximation to the data
 - E.g., lowess

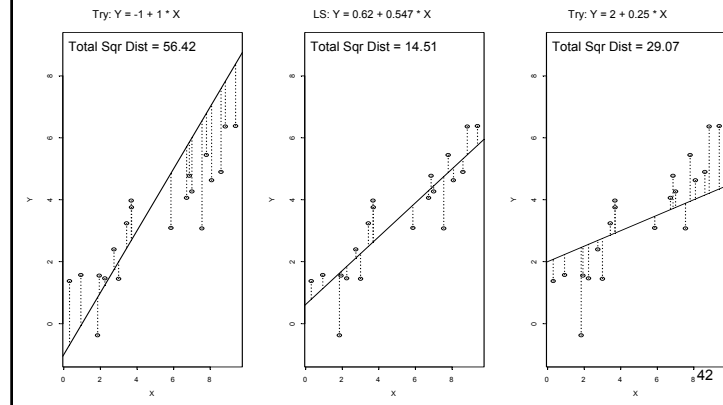
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Least Squares Line

- Find the straight line that minimizes total squared vertical distance from data to line
 - Conceptually: Trial and error search
 - Guess a formula for a line
 - Compute total squared distance from data to line
 - Iterate until smallest number found
 - Calculus:
 - Find a formula based on derivatives
 - Real life:
 - Computers find such estimates easily

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Conceptual Example



Stata: Superimposed Lines

- Basic Stata bivariate graph command
 - “`twoway ...`”
 - Special cases
 - “`twoway scatter ...`” (scatterplot of points)
 - “`twoway line ...`” (connect with lines)
 - “`twoway lfit ...`” (least squares fit)
 - “`twoway lowess ...`” (lowess curve)
- Superimposed graphs
 - `twoway (graphtyp ...) (graphtyp ...) ...`

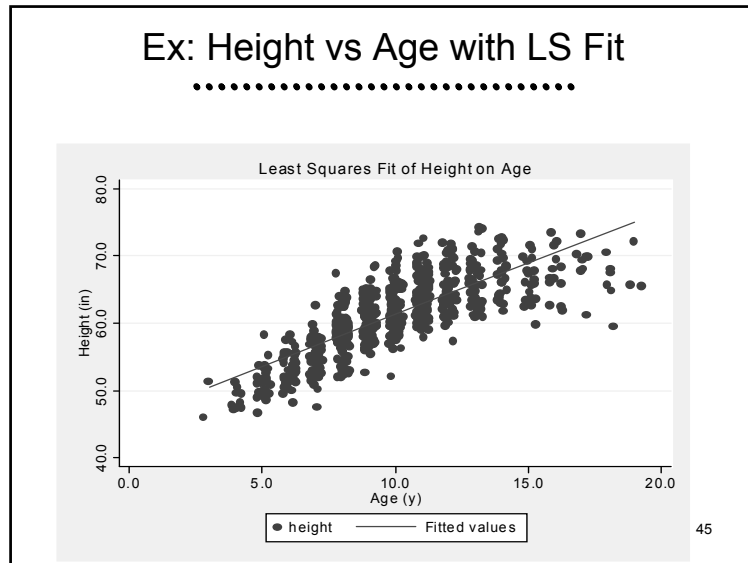
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Ex: Height vs Age with LS Fit

- Stata commands (all on same line)


```
twoway (scatter height age, jitter(3))
      (lfit height age),
      xtitle("Age (y)") ytitle("Height (in)")
      t1("Least Squares Fit of Height on Age")
```

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Interpretation

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- Clearly increasing trend in data

- Our eye tends to like to detect lines, so it takes careful inspection to decide a line is not the best fit
 - Note that at lowest ages and highest ages most data tend to be on one side of line rather than symmetric about line
 - Possible curvilinear association

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Lowess Smooths

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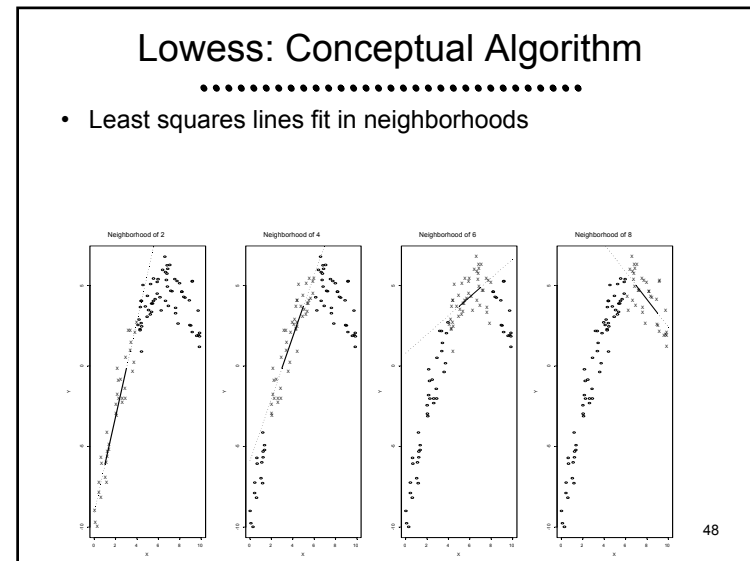
- Locally Weighted Scatterplot Smoother

- A smoother to find a smooth curve approximating relationship in the data

- For every value of X, fits straight lines in a neighborhood of that value
 - “Bandwidth” is width of window defining neighborhood
 - Weights closer data more heavily

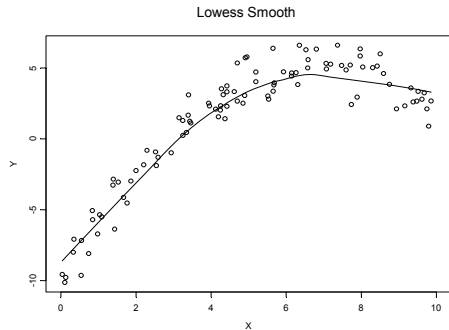
- Combines the estimates from different regions to form a smooth curve

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Lowess Smooth

- Combines locally fit least squares lines



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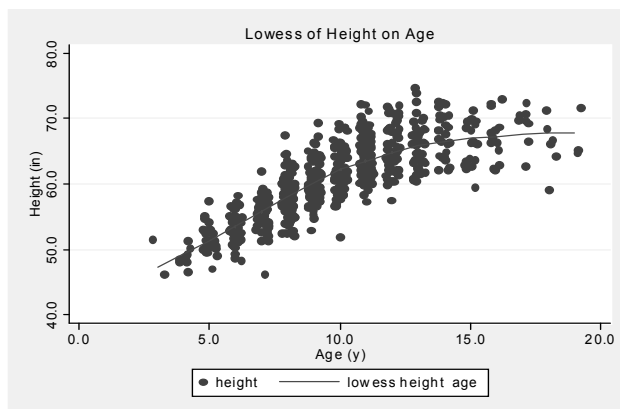
Ex: Height vs Age with Lowess

- Stata commands (all on one line)

```
twoway (scatter height age, jitter(3))
      (lowess height age),
      xtitle("Age (y)") ytitle("Height (in)")
      tl("Lowess of Height on Age")
```

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Ex: Height vs Age with Lowess



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Changing the Bandwidth

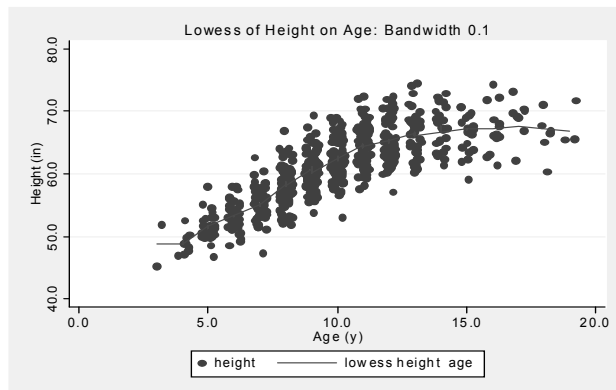
- Default bandwidth is 0.8 (80% of data)
 - I typically use the default of whatever program I am using
- Stata commands for less smoothing

```
twoway (scatter height age, jitter(3))
      (lowess height age, bwidth(.1)),
      xtitle("Age (y)") ytitle("Height (in)")
      tl("Lowess of Height on Age: Bandwidth 0.1")
```

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Changing the Bandwidth

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Ex: Showing Both LS, Lowess

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- Stata commands
 - Note specification of colors with option `col()`
 - Note specification of line patterns with option `lpattern()`

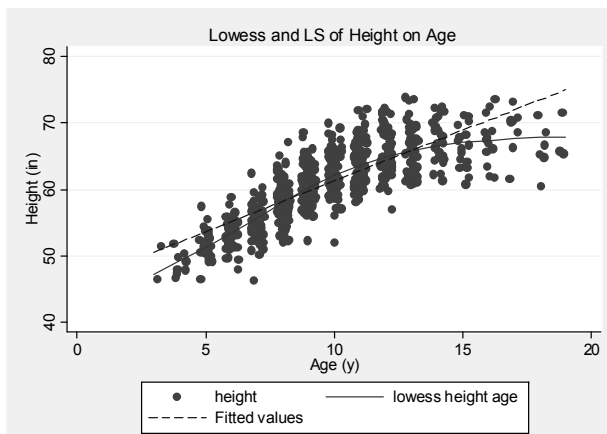
```

twoway (scatter height age, jitter(3))
      (lowess height age, col(red) lpattern(solid))
      (lfit height age, col(blue) lpattern(dash)),
      xtitle(Age (y)) ytitle(Height (in))
      tl(Lowess and LS of Height on Age)
    
```

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Ex: Showing Both LS, Lowess

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Interpretation

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- Lowess smooth shows that height tends to increase pretty linearly with age up until about age 11 or 12
- Height levels off in late teens with little change in mean height

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Other Smoothers

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- Many different methods of smoothing data have been proposed

- Lowess is often criticized due to the way it can accentuate data near the end of its range
 - One should not make too much of the way the estimate curve wiggles at the extremes of the data

- For my purposes, almost any smoother will do
 - I just want to have something that is not forced to be a line, and something that I did not draw
 - I can be just as biased as anyone

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Correlation

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Correlation Coefficient

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- A measure of the tendency of the largest measurements for one variable to be associated with the largest measurements of the other variable
 - Dimensionless
 - The sample correlation r estimates the population correlation ρ (rho)

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Pearson's Correlation Coefficient

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- Definition of correlation between X and Y:

$$\begin{aligned}
 r &= \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X) \text{Var}(Y)}} = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2} \sqrt{\sum_{i=1}^n (Y_i - \bar{Y})^2}} \\
 &= \frac{\sum_{i=1}^n X_i Y_i - n\bar{X}\bar{Y}}{\sqrt{\sum_{i=1}^n X_i^2 - n\bar{X}^2} \sqrt{\sum_{i=1}^n Y_i^2 - n\bar{Y}^2}}
 \end{aligned}$$

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Possible Values of r

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- Range of r : $-1 \leq r \leq 1$
- $r = 1$: perfect positive correlation
 - a graph of X vs Y will be a straight line with positive slope
- $r = -1$: perfect negative correlation
 - a graph of X vs Y will be a straight line with negative slope
- $r = 0$: no correlation

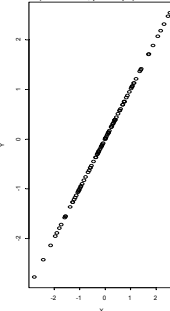
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Straight Line Relationships

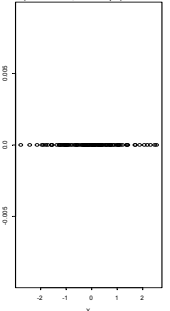
.....

- Pearson's correlation coefficient with linear data

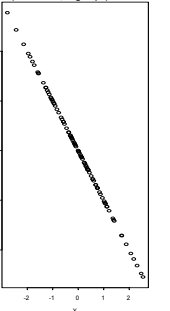
$r = 1.0$
(Perfect line, pos slope)



$r = 0.0$
(Perfect line, zero slope)



$r = -1.0$
(Perfect line, neg slope)



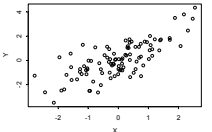
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Linear Trends in Data

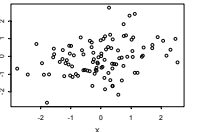
.....

- Pearson's correlation coefficient with variable data

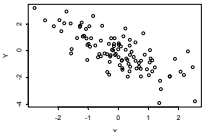
$r = 0.75$



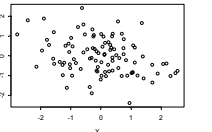
$r = 0.30$



$r = -0.75$



$r = -0.30$



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Correlation and Independence

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- Independent variables will have $\rho = 0$
 - (and r tending to be close to 0)
- However, uncorrelated variables are not necessarily independent
 - Correlation measures linear trend in the mean of one variable in groups defined by the other
 - It is possible that a nonlinear association exists between two variables, and that the first order trend is a zero slope

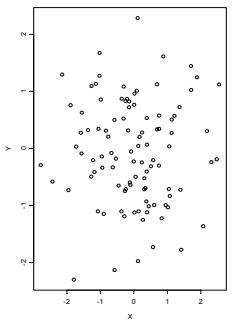
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Uncorrelated Variables

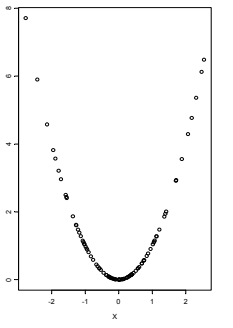
.....

- No linear trend between the variables

$r = 0.0$ (Independence)



$r = 0.0$ (Association, but no linear trend)



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Stata Commands

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- "correlate varlist"
 - Correlation of all pairs of variables
 - Missing data deleted on a casewise basis
- "pwcorr varlist"
 - Correlation of all pairs of variables
 - Missing data deleted on a pairwise basis

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Ex: Correlation in FEV Data

.....

```

. corr subjid age fev height sex smoke
(obs=654)
-----+-----
| subjid   age   fev  height   sex   smoke
-----+-----
subjid | 1.0000
    age | -0.0112  1.0000
    fev | -0.0147  0.7565  1.0000
  height | -0.0317  0.7919  0.8681  1.0000
    sex |  0.0407 -0.0291 -0.2084 -0.1590  1.0000
   smoke | -0.0601 -0.4043 -0.2454 -0.2804 -0.0756  1.0000
    
```

- Some of these correlations don't make much sense
 - subjid is a nominal variable
 - sex, smoke are binary variables

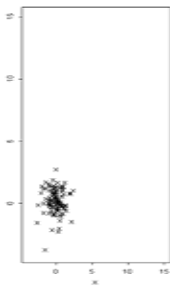
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Effect of Outliers on r

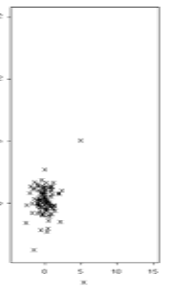
.....

- Pearson's correlation coefficient can be greatly affected by outliers

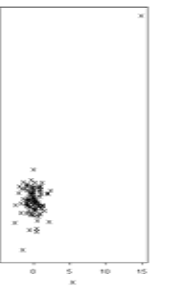
$r = 0.0$



$r = 0.2$



$r = 0.7$



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Spearman's Rank Correlation

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- To decrease the influence of outliers, Spearman's rank correlation coefficient computes the correlation of the ranks of the data
- In the previous example, the rank correlation is always the same: approximately 0.07

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Stata: Spearman's Correlation

.....

- `"spearman var1 var2"`
 - Correlation of one pair of variables
 - Cases with missing data for either variable are deleted, and then ranks are computed

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Ex: Correlation in PSA Data

.....

```
corr nadir pretx
(obs=43)
      |      nadir      pretx
-----+-----
nadir|      1.0000
pretx|      0.5371      1.0000

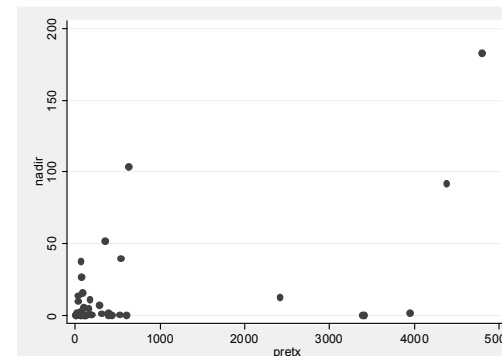
spearman nadir pretx
Number of obs =      43
Spearman's rho =      0.1489
```

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Ex: Nadir vs Pretreatment PSA

.....

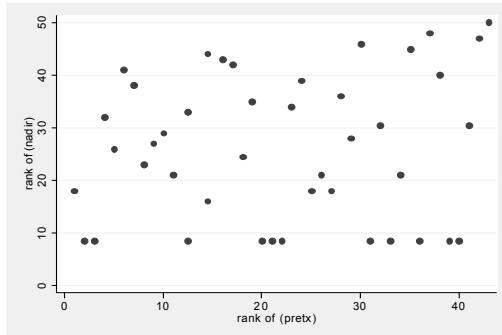
- Scatterplot of nadir versus pretx
 - `scatter nadir pretx`



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Ex: Nadir vs Pretx Ranks

- `egen rnknadir = rank(nadir)`
- `egen rnkpretx = rank(pretx)`
- `scatter rnknadir rnkpretx`



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Ex: Spearman's Corr vs r

- Possible explanation for lower rank correlation with Spearman's
 - Perhaps outliers in distribution of nadir and/or pretx unduly inflate r
 - Perhaps transforming to ranks masks true linear association in skewed variables

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Uses of Correlation

- By type of variable
 - Correlation is a mean, thus only makes sense when a mean does
 - Limited interpretability with categorical data
 - Of no scientific relevance with censored data
- By scientific question
 - Greatest relevance when looking for associations between variables
 - But not particularly generalizable across studies

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Correlation and Regression

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More Interpretable Formula for r

.....

$$r \approx \beta \sqrt{\frac{\text{Var}(X)}{\beta^2 \text{Var}(X) + \text{Var}(Y | X = x)}}$$

β = (LS) slope between Y and X

$\text{Var}(X)$ = variance of X in sample

$\text{Var}(Y | X = x)$ = variance of Y in groups that have same value of X

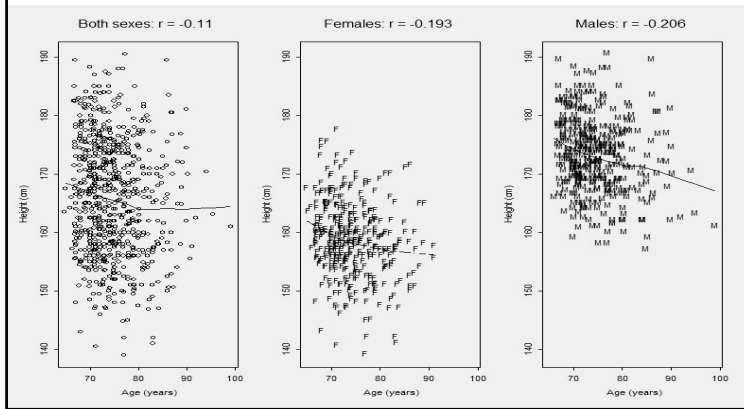
Properties of Correlation

.....

- Correlation tends to increase in absolute value as
 - The absolute value of the slope of the line increases
 - The variance of data decreases within groups that share a common value of X
 - The variance of X increases
 - (Sample size is unimportant in tendencies toward lower or higher correlation)

Ex: Height vs Age (by Sex)

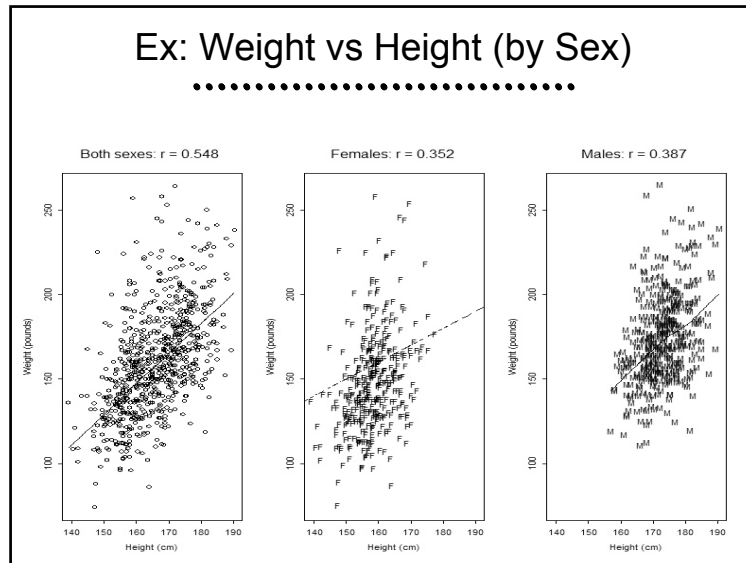
.....



Ex: Height vs Age (by Sex)

.....

- Correlation between Height and Age
 - Males: $r = -0.206$; Females: $r = -0.193$
 - Combined: $r = -0.110$
- Less extreme r in combined sexes
 - Approximately same slope in each sex and overall
 - Approximately same variance of age in each sex and overall
 - Combined group has higher within group variance of height by age (due to sex effect)



- ### Ex: Weight vs Height (by Sex)
-
- Correlation between Height and Weight
 - Males: $r = .387$; Females: $r = 0.352$
 - Combined: $r = 0.548$
 - More extreme r in combined sexes
 - Approximately same slope in each sex and overall
 - Approximately same within group variance (by height) for each sex and overall
 - Combined group has higher variance of height
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- ### Scientific Relevance of r
-
- It should be noted that
 - the slope between X and Y is of scientific interest
 - the variance of $Y|X=x$ is partly of scientific interest, but it can be affected by restricting sampling to certain values of another variable
 - E.g., $\text{var}(\text{Height} | \text{Age})$ is less in males than when both sexes are included
 - the variance of X is often set by study design
 - This is often not of scientific interest
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