Biost 517
Applied Biostatistics I

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Lecture 4:
Descriptive Measures of Location

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Lecture Outline

- Notation
- Univariate Measures of Location
  - Mode
  - Means (Arithmetic, Geometric, Harmonic)
  - Median (Other Quantiles)
  - Proportions
  - Odds

Notation

Random Variables

- Variables that take on particular values according to a probability distribution
  - The value of the random variables may be a label (sometimes a numeral is used as a label) or a number
  - Mathematicians tend to use X, Y, Z
    - With no scientific problem in mind, I will too
    - For a particular scientific problem, I will tend to use mnemonics, e.g., AGE, HEIGHT, WEIGHT

Sample of Measurements

- Measurements made on n subjects
  - Subscripts on a random variable denote the measurements made on different subjects
    - X_1 will be the measurement of X on subject 1
    - X_5 will be the measurement of X on subject 5
    - X_i will be the measurement of X on subject i
  - As a general rule, it is arbitrary which subject is the first, second, etc. so long as we are consistent

Random Variables vs Observed

- The notation can seem a little confusing:
  - Random variable *Chol* might denote the idea of making a measurement of cholesterol
  - Random variable $Chol_i$ might denote the idea of making a measurement on the i-th subject
  - Constant *c* might denote a particular value of the random variable *Chol*
  - c_i might denote the particular value observed for the measurement on the i-th subject

Notation for Ordered Measurements

- Some descriptive statistics make use of the order statistics for a sample:

$$\text{Data} \quad \{X_1, X_2, \ldots, X_n\}$$

$$\text{Order stats} \quad \{X_{(1)}, X_{(2)}, \ldots, X_{(n)}\}$$

$$(X_{(1)} \le X_{(2)} \le \cdots \le X_{(n)} \text{ are the data in order})$$

Univariate Measures of Location

Measures of Central Tendency

- In order (more or less) of usefulness
  - Mean (average; arithmetic mean)
  - Geometric mean
  - Median
  - Proportion exceeding a specific threshold
  - Odds of exceeding a specific threshold
  - Mode

"You better think (think), think about what you're trying to do…"

Aretha Franklin, "Think"

Purpose of Descriptive Statistics

- Identify errors in measurement, data collection
- Characterize materials and methods
- Assess validity of assumptions needed for analysis
- Straightforward estimates to address scientific question
- Hypothesis generation

Identify Type of Measurement

- The way in which a variable is measured will affect the descriptive statistics that are of interest
  - Binary (dichotomous, Bernoulli)
  - Nominal (unordered categorical)
  - Ordered categorical
  - Quantitative
    - Discrete, interval continuous, ratio continuous
  - Censored

(Arithmetic) Mean

- Definition of sample arithmetic mean:
  - Sum of measurements divided by the number of measurements (average)
  - Notation: Usually denoted by a bar over the variable

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i = \frac{(X_1 + X_2 + \cdots + X_n)}{n}$$

Mean: Example

- Data: {1, 3, 6, 3, 2, 3, 6, 7, 1, 1}
  - Number of observations: 10
  - Sum of observations: 33
  - Arithmetic mean: 3.3

Mean: Interpretation

- The "point of balance" for the distribution
  - Center of gravity
- If all measurements were the same, the arithmetic mean is the value that they would all be in order to have the same total
- Allows prediction of the total of an arbitrary number of observations
  - Applications in gambling: How much would I expect to win/lose if I play 1,000 times
  - Similarly useful in health care cost analysis

Mean: Types of Variables

- Types of variables
  - Defined only for variables that take on numeric values (sum must make sense)
  - Most sensible when differences have scientific interpretation on a constant scale
    - Treats observation of {1,5} similar to {3,3}
    - (5 - 4) should be similar equivalent to (3 - 2), etc.
    - (But see comments regarding comparisons of ordered categorical variables)

Mean: Censored Variables

- Not of interest with variables measuring censored times to an event
  - Must know all relevant values exactly in order to compute sum
    - The observation time is a mixture of times to event and times to censoring
    - The indicator of events is measured over varying time periods
  - (Alternative methods using Kaplan-Meier estimate of survivor function (see later) sometimes do allow estimation of mean with censored data)

Mean: Descriptive Uses

- By purpose of descriptive statistics
  - Characterizing sample
    - Often used as a "typical value"
    - Note that mean is heavily influenced by large outliers, so sometimes mean does not reflect the quantity desired
  - Assessing validity of assumptions
    - Often we model the mean by assuming linear, quadratic, etc. relationship
    - Most often best measure of potential confounding

Mean: Scientific Uses

- Prediction of an individual observation
  - (minimizes squared error loss)
- Clustering:
  - Some algorithms minimize distance to means
- Quantifying distributions:
  - Sometimes best related to the scientific question
    - E.g.: Mean blood pressure and chronic vascular disease
    - E.g.: Studying total health care costs
- Comparing distributions:
  - Sensitive to a wide variety of differences in distn

Mean:Ordered Categorical Data

- The mean by itself is not scientifically interpretable
- The mean can still detect differences in the distributions
  - Sensitive to certain tendencies for higher measurements in one group

Geometric Mean

- Definition of geometric mean:
  - Exponentiated arithmetic mean of log transformed data
    - (exponentiation and logarithm should be same base)

$$\exp\left(\frac{1}{n}\sum_{i=1}^{n}\log\left(X_i\right)\right) = \sqrt[n]{\prod_{i=1}^{n} X_i}$$

Geom Mean: Descriptive Uses

- The geometric mean is related to the mean of log transformed data
  - All comments about the arithmetic mean need to apply to the log transformed data
- When variables are measured on a exponential scale, geometric means tend to be more stable
  - Serial dilutions used for measuring titers
- Ratios of geometric means tend to be more stable than ratios of arithmetic means.
  - The log of a ratio is the difference of logs

Geom Mean: Scientific Uses

- Sometimes a better measure of “typical” values for skewed data (with large outliers)
- Greater statistical precision than mean when standard deviations are proportional to mean

Harmonic Mean

- Definition of harmonic mean
  - Reciprocal of the arithmetic mean of the reciprocals of the data

$$\frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{X_i}}$$

Harmonic Means: Uses

- The harmonic mean is related to the mean of the reciprocal of the data
  - All comments about the arithmetic mean need to apply to the reciprocal of the data
- The harmonic mean sometimes has scientific interpretation
  - E.g., in electricity, resistance of parallel resistors
  - E.g., in studying vascular flow and blood pressure

Median

- Definition:
  - The value that is larger than half the population and smaller than half the population

$$mdn(X) \text{ is any value } M \text{ such that}$$
$$\Pr(X \le M) \ge 0.5$$
$$\Pr(X \ge M) \ge 0.5$$

Calculation of Sample Median

- In order to find a unique sample median, we usually use the following definition for uncensored data

$$n \text{ odd}: mdn(X) = X_{\left(\frac{n+1}{2}\right)}$$
$$n \text{ even}: mdn(X) = \frac{1}{2}\left(X_{\left(\frac{n}{2}\right)} + X_{\left(\frac{n}{2}+1\right)}\right)$$

Median: Examples

- Example (odd number of observations)
  - Data: {1, 3, 6, 3, 2, 3, 6, 7, 1, 1, 5}
  - Order statistics: {1, 1, 1, 2, 3, 3, 3, 5, 6, 6, 7}
  - Median: 3
- Example (even number of observations)
  - Data: {1, 6, 2, 3, 6, 7, 1, 5}
  - Order statistics: {1, 1, 2, 3, 5, 6, 6, 7}
  - Median: 4

Median: Type of Variables

- Concept defined for any ordered variable
- Special methods (Kaplan-Meier estimates) must be used with censored data
  - The sample median is not of interest
    - The observation time is a mixture of times to event and times to censoring
    - The indicator of events is measured over varying time periods
  - More often able to estimate median from censored data than mean

Median: Descriptive Uses

- Characterizing distribution of sample
  - "Typical" value, especially when number of observations above or below is most meaningful scientifically
  - Less influenced (not influenced) by outliers

Median: Scientific Uses

- Prediction: If median is "better behaved"
  - (tends to minimize absolute error loss)
- Quantifying distributions:
  - Not sensitive to "outliers"
  - Estimated more efficiently in presence of outliers
- Comparing distributions:
  - Not sensitive to "outliers"
    - Sometimes more precision for skewed data
    - Not as useful when distn only differ in the tails

Proportions

- Dichotomize observations by defining a group of interest
  - Ordered: According to whether they exceed some threshold
    - E.g., Age > 50
  - Unordered: Divide into groups
    - E.g., Marital status separated or divorced

$$Y_i = \begin{cases} 1 & \text{if in group of interest} \\ 0 & \text{otherwise} \end{cases}$$

Proportions

- Proportion of subjects in the group of interest is the mean of the dichotomized data

$$\hat{p} = \overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$$

Proportion: Type of variables

- Types of variables
  - Can be defined for nearly any variable
    - Dichotomization must be scientifically interpretable
    - Not of interest with variables measuring censored times to an event
      - The observation time is a mixture of times to event and times to censoring
      - The indicator of events is measured over varying time periods
    - Natural for binary variables

Proportion: Descriptive Uses

- By purpose of descriptive statistics
  - Useful in characterizing distribution
    - In presence of scientifically meaningful threshold
      - E.g., proportion of subjects with cholesterol below 200
    - When mode is minimum or maximum
      - E.g., proportion of nonsmokers

Proportion: Scientific Uses

- Scientific questions
  - Prediction: Discrimination, classification
    - Discrimination
  - Quantifying distributions:
    - Binary variables
    - Scientifically important thresholds, categories
  - Comparing distributions:
    - Binary variables
    - Scientifically important thresholds, categories

Odds

- Odds of being in group of interest:
  - Definition
    - Dichotomize observations as with proportion
    - Ratio of proportion to 1 minus proportion

$$\hat{p} = \overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$$

$$\hat{o} = \frac{\hat{p}}{1-\hat{p}}$$

Odds: Uses

- As with proportions, except
  - Odds is less easily understood by a lay person
    - Odds of rolling 1 on a die: 1/5
  - However, odds (actually log odds) is often more convenient in modeling due to greater range of possible values
    - proportion is between 0 and 1
    - odds is between 0 and infinity
    - log odds is any real number

Mode

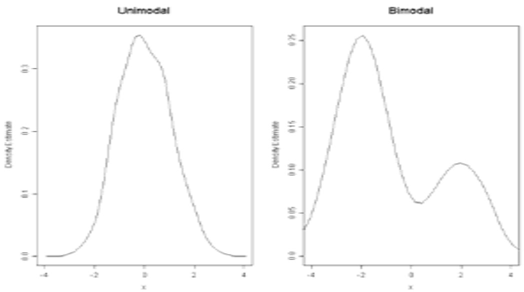
- Definition:
  - Discrete data: Most frequently occurring value
  - Continuous data: (Local) maximum of density
    - Determined from histogram or density

Mode: Discrete Data Examples

- Data: {1, 3, 7, 3, 2, 3, 6, 7}
  - Order statistics: {1, 2, 3, 3, 3, 6, 7, 7}
  - Mode: 3
- Data: {1, 3, 6, 3, 2, 3, 6, 7, 1, 1}
  - Order statistics: {1, 1, 1, 2, 3, 3, 3, 6, 6, 7}
  - Modes: 1, 3
- Data: {1, 0, 9, -3, 2, 3, 6, 7, 8, 4}
  - Order statistics: {-3, 0, 1, 2, 3, 4, 6, 7, 8, 9}
  - Modes: none (or all?)

Mode: Continuous Data Example

– Density estimates from samples



Mode: Types of Variables

- Types of variables
  - Defined for both categorical and continuous
    - (though definitions differ)
  - Not dependent on ability to order values
  - Not of interest with variables measuring censored times to an event
    - The observation time is a mixture of times to event and times to censoring
    - The indicator of events is measured over varying time periods

Mode: Uses

- By purpose of descriptive statistics
  - Sometimes useful in characterizing sample
    - Suitable as a “typical” value, providing it has high enough frequency or density
  - Hypothesis generation
    - Multimodal distributions might indicate mixture of populations
      - Subgroups of subjects might behave differently

Mode: Scientific Uses

- Prediction:
  - If mode represents overwhelming majority of distribution
- Cluster analysis:
  - Multimodal distributions often considered mixture of populations
- Quantifying or comparing distributions:
  - Rarely used due to difficult inference

Stata Commands

Stata: `describe`

- "`describe` *varlist*"
  - Provides type of variable
  - Value labels

Stata: `inspect`

- "`inspect` *varlist*"
  - Line printer histogram
  - Counts of missing vs nonmissing
  - Counts of integer vs noninteger
  - Counts of negative, zero, positive values

Stata: `summarize`

- "`summarize` *varlist,* `format`"
  - Table format with number of nonmissing observations, mean, standard deviation, minimum maximum
- "`summarize` *varlist*`, detail format`"
  - Additionally provides quantiles, skewness, kurtosis, but not in useful table

Stata: `means`

- "`means` *varlist*"
  - Table format with arithmetic, geometric, and harmonic means
  - Also gives
    - number of nonmissing observations
    - confidence intervals for infererence)
- (Could also obtain geometric mean by log transforming data, taking mean, exponentiating)

Stata: `centile`

- "`centile` *varlist,* *`cen(20 25 50)`*"
  - Table format with requested quantiles
  - Also gives
    - confidence intervals for infererence
  - 25^{th}, 75^{th} percentiles may not agree with other Stata functions

Stata: `tabstat`

- "`tabstat` *varlist,* `stat(n mean sd min p25 med p75 max) col(stat) format`"
  - Table format with univariate descriptive statistics that I like best

Stata: `tabulate`

- "`tabulate` *var*"
  - Provides frequency for each value of a single variable (counts, proportions, cumulative proportion)

Stata: Dichotomizing data

- Create a new variable using “`generate`” and “`replace`”
- Example: Dichotomize age at 9 and older
  - `g age9over= 0`
  - `replace age9over= 1 if age>=9`

Stata: Computing the Mode

- Sample Mode: Discrete data
  - “table *var*”
    - Examine (possibly lengthy) output for highest frequency
- Mode of density for continuous data
  - Graphical display using “kdensity *var*”
    - Better labeling of axes with options “xlabel”, “ylabel”
  - Numerical output by examining generated variables “kdensity *var*, g(*x d*)” for the value of *x* that corresponds to maximal value for *d*