Diagnostic Testing in Medicine

Sensitivity and Specificity

We most often characterize the sensitivity and specificity of a diagnostic test

Sensitivity of test: Probability of positive in diseased

- Sample a cohort of subjects with the disease
- Estimate the proportion who have a positive test result: Pr (+ | D)

Specificity of test: Probability of negative in healthy

- Sample a cohort of healthy subjects
- Estimate the proportion who have a negative test result: Pr(- | H)

Predictive Values of Positive and Negative

We are actually interested in the diagnostic utility of the test: Predictive value of positive and negative

Predictive value of a positive test: Probability of disease when test is positive

- Pr (D | +)

Predictive value of a negative test: Probability of health when test is negative

- Pr (H | -)

Bayes Rule for Binary Random Variables

We usually compute the predictive value of positive and negative tests using Bayes rule

$$\Pr(D \mid +) = \frac{\Pr(+ \mid D)\Pr(D)}{\Pr(+ \mid D)\Pr(D) + \Pr(+ \mid H)\Pr(H)}$$

$$\Pr(H \mid -) = \frac{\Pr(- \mid H)\Pr(H)}{\Pr(- \mid H)\Pr(H) + \Pr(- \mid D)\Pr(D)}$$

Role of Prevalence

Key property: Computation of predictive value of positive uses sensitivity, specificity, AND prevalence of disease

$$\Pr(D \mid +) = \frac{\Pr(+ \mid D)\Pr(D)}{\Pr(+ \mid D)\Pr(D) + \Pr(+ \mid H)\Pr(H)}$$

$$PVP = \frac{Sens \times Prev}{Sens \times Prev + (1 - Spec) \times (1 - Prev)}$$

Role of Prevalence

Key property: Computation of predictive value of negative uses sensitivity, specificity, AND prevalence of disease

$$\Pr(H \mid -) = \frac{\Pr(- \mid H)\Pr(H)}{\Pr(- \mid H)\Pr(H) + \Pr(- \mid D)\Pr(D)}$$

$$PVN = \frac{Spec \times (1 - Prev)}{Spec \times (1 - Prev) + (1 - Sens) \times Prev}$$

Diagnostic Testing: Example

VDRL in diagnosing syphilis: High sensitivity and high specificity

Sensitivity of test: Probability of positive in diseased

- 90% of subjects with syphilis test positive
- (Actually depends on duration of infection)

Specificity of test: Probability of negative in healthy

- 98% of subjects without syphilis test negative
- (Actually depends on age and prevalence of certain other diseases)

Diagnostic Testing: Example

Predictive values when prevalence is high

Ex: STD clinic

- Prevalence of syphilis 30%
- PV+: 95% with positive VDRL have syphilis

		VDRL Pos	VDRL Neg	Tot
Syphilis	Yes	270	30	300
	No	14	686	700
Total		284	716	1000

Diagnostic Testing: Example

Predictive values when prevalence is low

Ex: Screening for marriage exam

- Prevalence of syphilis 2%
- PV+: 48% with positive VDRL have syphilis

		VDRL		
		Pos	Neg	Tot
Syphilis	Yes	18	2	20
	No	20	960	980
Total		38	962	1000

Role of Prevalence

Bottom line:

Predictive value of a diagnostic test depends heavily on the prevalence of the disease

More generally:

- When using Bayes rule, to calculate probabilities, the computed values are specific to the assumed "prior" information